EE331—EXAMPLE #1: COMPLEX NUMBERS AND PHASORS

1. Evaluate and simplify the following complex expression into rectangular form:

$$z = e^{-j\pi/4}\left[\frac{1}{\sqrt{-1}}(a+ja)(a+ja)^* e^{j\pi}\right]^{1/2}$$

First consider the expression inside the bracketed square root function. We know that

$$\frac{1}{\sqrt{-1}} = \frac{1}{j} = -j$$

and

$$(a+ja)(a+ja)^* = (a+ja)(a-ja) = a^2 + a^2 = 2a^2$$

Because a square root is involved, it's best to use polar form. We convert $-j$ to polar form using

$$-j = e^{-j\pi/2}$$

Aside: Show this is true using Euler's rule.

$$e^{-j\pi/2} = \cos(\pi/2) - j\sin(\pi/2) = 0 - j = -j$$

Thus, we have

$$\begin{aligned}\frac{1}{\sqrt{-1}}(a+ja)(a+ja)^* e^{j\pi} &= e^{-j\pi/2} 2a^2 e^{j\pi} \\ &= 2a^2 e^{j\pi/2}\end{aligned}$$

Now take the square root:

$$\left[2a^2 e^{j\pi/2}\right]^{1/2} = \pm\sqrt{2}\,a\,e^{j\pi/4}$$

Finally,

$$\begin{aligned}z &= e^{-j\pi/4}\left(\pm\sqrt{2}\,a\,e^{j\pi/4}\right) \\ &= \pm\sqrt{2}\,a\end{aligned}$$

2. Convert the following phasor to its instantaneous (time) form:

$$V_s(z) = 10e^{-(2+j3)z+j\pi/6} \;\; [\mathrm{V}]$$

First multiply the phasor by $e^{j\omega t}$ and take the real part:

$$\begin{aligned} V(z,t) &= \mathcal{R}e\left[V_s e^{j\omega t}\right] \\ &= \mathcal{R}e\left[10e^{-(2+j3)z+j\pi/6+j\omega t}\right] \\ &= \mathcal{R}e\left[10e^{-2z}e^{j(\omega t-3z+\pi/6)}\right] \end{aligned}$$

Now apply Euler's rule:

$$\begin{aligned} V(z,t) &= \mathcal{R}e\left[10e^{-2z}\left(\cos\left(\omega t-3z+\pi/6\right)+j\sin\left(\omega t-3z+\pi/6\right)\right)\right] \\ &= 10e^{-2z}\cos\left(\omega t-3z+\pi/6\right) \;\; [\mathrm{V}] \end{aligned}$$