A 50-Ω transmission line is connected to an antenna with impedance $Z_L = 25 - j50\ \Omega$. Find (a) the two possible positions for the stubs (d_A and d_B) and (b) their respective lengths (l_A and l_B) to match the antenna load to the line.

(a) First find the normalized load impedance z_L.

$$z_L = \frac{Z_L}{Z_0} = \frac{25 - j50}{50} = 0.5 - j$$

Locate z_L on the Smith chart and draw the SWR circle. Draw a line from z_L through the origin and back out through the SWR circle. The point of intersection is the normalized admittance y_L (which we also could have calculated, but it's faster to use the Smith chart).

$$y_L = \frac{1}{z_L} = 0.4 + j0.8$$

Now locate the points where the SWR circle intersects the $g = 1$ circle. (Recall that the g and y circles are just the r and x circles, respectively.) Draw lines from the origin through these points and label them A and B. At A:

$$y_{d_A} = 1 + j1.6$$

and at B:

$$y_{d_B} = 1 - j1.6$$

Recall that we want y_{in} to be 1:

$$y_{in} = y_d + y_s = 1$$

Thus, we need

$$y_{s_A} = -j1.6$$

and

$$y_{s_B} = j1.6$$

Next measure the distances from y_L to points A and B, moving toward the generator (you're finding the distances from the load where you can place a shunt stub tuner). These distances are measured in wavelengths:

$$d_A = 0.178\lambda - 0.116\lambda = 0.062\lambda$$

$$d_B = 0.322\lambda - 0.116\lambda = 0.206\lambda$$

Note that these are the two possible *distances* (begins with the letter *d*) from the load where you can put the stub tuner so that the real part of y_{in} is 1 because that's what will match the load to the line. Now we need to cancel out the imaginary part of y_d to make y_{in} purely real (and equal to 1).

(b) Next find the *length* (begins with the letter *l*) of each shunt stub tuner such that the imaginary part of y_{d_A} and y_{d_B} are cancelled by the admittance of the respective stub tuner. (Recall that the impedance of a short-circuited or open-circuited transmission line is totally reactive—i.e., imaginary.)

Note that this is a new line that is unrelated to the old line. It has its own characteristic impedance and load impedance. The load impedance of a short-circuited line is $z_L = 0$; the load admittance is the reciprocal of this which means it's infinite. The point on the Smith chart where the admittance is infinite is (1,0), i.e., it's the same point where the impedance is zero. Mark this point on the Smith chart. Measure the length of the first stub starting from this infinite load admittance and moving toward the generator until you reach the point where $y_{s_A} = -j1.6$. This gives the length l_A.

$$l_A = 0.339\lambda - 0.25\lambda = 0.089\lambda$$

Now move from the infinite load admittance toward the generator until you reach the point where $y_{s_B} = j1.6$. This gives the length l_B.

$$l_B = (0.161\lambda - 0) + (0.5\lambda - 0.25\lambda) = 0.411\lambda$$

(or you could have just added 0.161λ and 0.25λ).

So in summary, you've found two different designs to match the load to the line using a shunt stub tuner:

- Design #1: Place a shorted stub of length 0.089λ a distance 0.062λ from the load.
- Design #2: Place a shorted stub of length 0.411λ a distance 0.206λ from the load.

The Complete Smith Chart

Black Magic Design