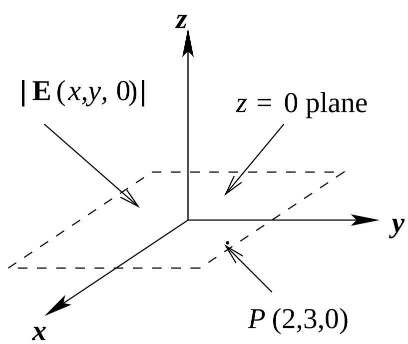
A vector field is given by $\mathbf{E}(x,y,z) = 24xy\hat{\mathbf{a}}_x + 12x^2\hat{\mathbf{a}}_y + 18z^2\hat{\mathbf{a}}_z$ V/m. Find (a) the magnitude of $\mathbf{E}$ in the plane $(x,y,0)$—i.e., the $z=0$ plane, (b) the value of $\mathbf{E}$ along the line $(0,4,z)$, and (c) the vector component of $\mathbf{E}$ parallel to the x axis. (d) Plot $|\mathbf{E}|$ along the line $(0,4,z)$ for $-2 \leq z \leq 2$.

(a) You're given the vector field $\mathbf{E}(x,y,z)$ which is a function of x, y, and z. Its magnitude and direction vary everywhere in space. In part (a) we just want to know the magnitude of $\mathbf{E}$ in the $z=0$ plane. It will vary in this plane depending on what point in the plane we're considering. For example, at $x=2$ and $y=3$, the magnitude of $\mathbf{E}$ is:



$$\begin{aligned}|\mathbf{E}(2,3,0)| &= |24(2)(3)\hat{\mathbf{a}}_x + 12(2)^2\hat{\mathbf{a}}_y + 18(0)^2\hat{\mathbf{a}}_z| \\ &= |144\hat{\mathbf{a}}_x + 48\hat{\mathbf{a}}_y| \\ &= \sqrt{(144)^2 + (48)^2} \\ &= 151.7893 \text{ V/m}\end{aligned}$$

We know $\mathbf{E}$ everywhere in space. It's:

$$\mathbf{E}(x,y,z) = 24xy\hat{\mathbf{a}}_x + 12x^2\hat{\mathbf{a}}_y + 18z^2\hat{\mathbf{a}}_z \text{ V//m}$$

In the $z=0$ plane, it's:

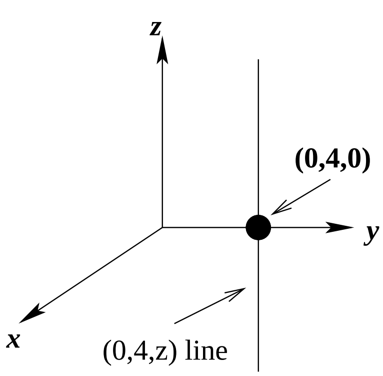
$$\mathbf{E}(x,y,0) = 24xy\hat{\mathbf{a}}_x + 12x^2\hat{\mathbf{a}}_y \text{ V/m}$$

Hence, its magnitude in the $z=0$ plane is:

$$|\mathbf{E}(x,y,0)| = \sqrt{(24xy)^2 + (12x^2)^2} = 12|x|\sqrt{x^2+4y^2} \text{ V/m}$$

Note the absolute value! Question: What is $\mathbf{E}$ in the $y=2$ plane?

(b) Note that a line in space is designated by 2 fixed points and 1 variable. The line $(0,4,z)$ has $x=0$ and $y=4$.



The value of $\mathbf{E}$ along this line is simply:

$$\mathbf{E}(0,4,z) = 0\hat{\mathbf{a}}_x + 0\hat{\mathbf{a}}_y + 18z^2\hat{\mathbf{a}}_z = 18z^2\hat{\mathbf{a}}_z \text{ V/m}$$

Thus, $\mathbf{E}$ points in the positive z direction and has a magnitude of $18z^2$. Question: What is $\mathbf{E}$ at (0,4,0)?

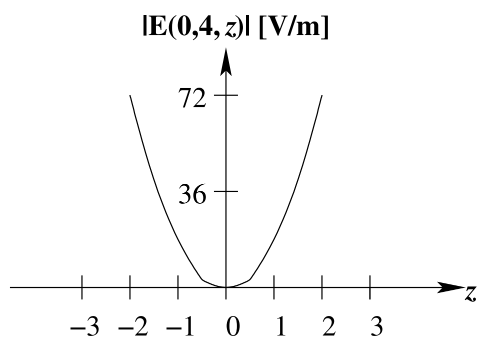
(c) Next we want to find the vector component of **E** parallel to the x axis. This is given by:

$$\begin{aligned}
\mathbf{E}_x = E_x\widehat{\mathbf{a}}_x &= (\mathbf{E}\cdot\widehat{\mathbf{a}}_x)\widehat{\mathbf{a}}_x \\
&= \left[\left(24xy\widehat{\mathbf{a}}_x + 12x^2\widehat{\mathbf{a}}_y + 18z^2\widehat{\mathbf{a}}_z\right)\cdot\widehat{\mathbf{a}}_x\right]\widehat{\mathbf{a}}_x \\
&= 24xy\widehat{\mathbf{a}}_x \text{ V/m}
\end{aligned}$$

(d) Finally, plot the magnitude of **E** along the line $(0, 4, z)$. First, note that we want to plot the magnitude which is a positive scalar function and **not a vector**. Second, note that we don't really know how to plot **E** itself because we don't know how to plot direction. Third, when we say "along the line," we mean we want to know the value of $|\mathbf{E}|$ as a function of z for a fixed $x = 0$ and $y = 4$. Thus,

$$|\mathbf{E}(0,4,z)| = \sqrt{(18z^2)^2} = 18z^2 \text{ V/m}$$

Plotting this gives:



What this says is that the value of $|\mathbf{E}|$ at $(0,4,0)$ is 0; at $(0,4,\pm1)$ it's 18 V/m; at $(0,4,\pm2)$ it's 72 V/m; and so on.