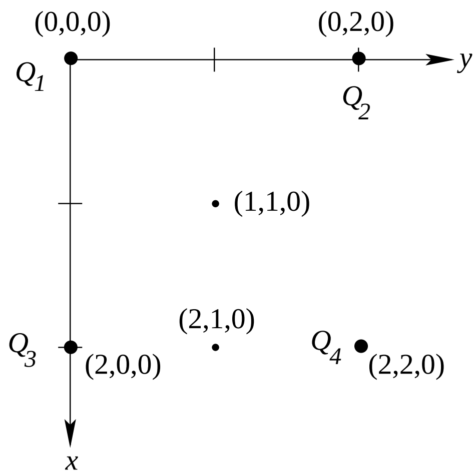
Four point charges are in the $z = 0$ plane at (0,0,0), (0,2,0), (2,0,0), and (2,2,0) in free space. If $Q_1 = Q_2 = Q_3 = Q_4 = 1\ \mu$C, (a) what is $\mathbf{E}(1,1,0)$? (b) Find **E** at (2,1,0).

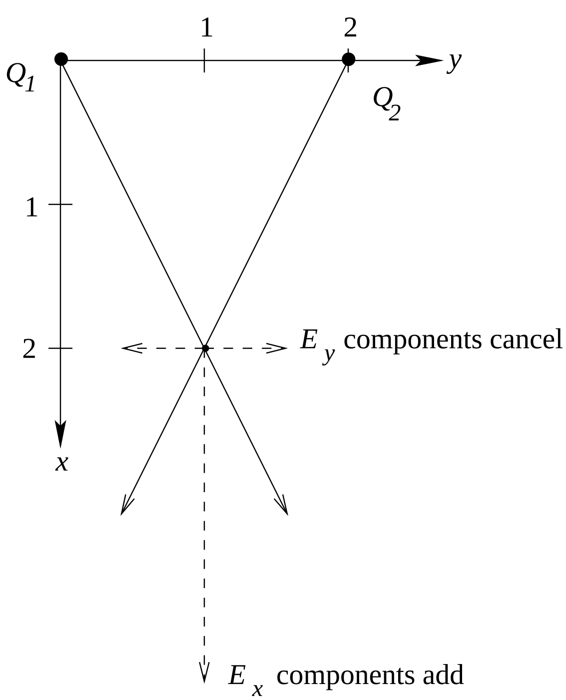
First make a sketch.



(a) Use superposition and symmetry: Q_1 and Q_4 cancel, and Q_2 and Q_3 cancel. Thus,

$$\mathbf{E}(1,1,0) = 0$$

(b) At (2,1,0), Q_3 and Q_4 cancel by superposition and symmetry, but what about Q_1 and Q_2?



$$\mathbf{E}(2,1,0) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{\mathbf{r}-\mathbf{r}_1'}{|\mathbf{r}-\mathbf{r}_1'|^3} + \frac{\mathbf{r}-\mathbf{r}_2'}{|\mathbf{r}-\mathbf{r}_2'|^3}\right)$$

$$\mathbf{r} = (2,1,0)$$
$$\mathbf{r}_1' = (0,0,0)$$
$$\mathbf{r}_2' = (0,2,0)$$
$$\mathbf{r}-\mathbf{r}_1' = (2,1,0)$$
$$\mathbf{r}-\mathbf{r}_2' = (2,-1,0)$$
$$|\mathbf{r}-\mathbf{r}_1'| = \sqrt{5}$$
$$|\mathbf{r}-\mathbf{r}_2'| = \sqrt{5}$$

$$\Rightarrow \mathbf{E}(2,1,0) = \frac{10^{-6}}{4\pi\varepsilon_0}\left(\frac{2\widehat{\mathbf{a}}_x}{(\sqrt{5})^3} + \frac{2\widehat{\mathbf{a}}_x}{(\sqrt{5})^3}\right)$$

$$= 3.2156\widehat{\mathbf{a}}_x \text{ kV/m}$$