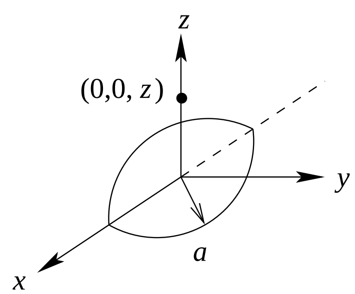
A circular ring of charge of radius = a m lies in the $z = 0$ plane and is centered at the origin. Assume ρ_L has a constant value ρ_0 and $\varepsilon = \varepsilon_0$. Find V at $(0, 0, z)$.

First make a sketch and choose the appropriate coordinate system.



Next write down the appropriate form of the equation for the electric potential. In this case we have a line charge density so we choose

$$V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho_L \, dl'}{|\mathbf{r} - \mathbf{r}'|} = \frac{\rho_0}{4\pi\varepsilon_0} \int \frac{dl'}{|\mathbf{r} - \mathbf{r}'|}$$

Next we identify $\mathbf{r}$, $\mathbf{r}'$, dl', $|\mathbf{r} - \mathbf{r}'|$, and the limits of integration:

$$\mathbf{r} = z\widehat{\mathbf{a}}_z \;\; [\mathrm{m}]$$

$$\mathbf{r}' = a\widehat{\mathbf{a}}_\rho \;\; [\mathrm{m}]$$

$$dl' = a\,d\phi' \;\; [\mathrm{m}]$$

$$|\mathbf{r} - \mathbf{r}'| = \sqrt{z^2 + a^2} \;\; [\mathrm{m}]$$

and the limits of integration are $0 \rightarrow 2\pi$. Thus,

$$V(0,0,z) = \frac{\rho_0}{4\pi\varepsilon_0} \int_0^{2\pi} \frac{a\,d\phi'}{\sqrt{z^2 + a^2}} = \frac{\rho_0 a}{4\pi\varepsilon_0\sqrt{z^2 + a^2}} \int_0^{2\pi} d\phi' = \frac{\rho_0 a}{2\varepsilon_0\sqrt{z^2 + a^2}} \;\; [\mathrm{V}]$$