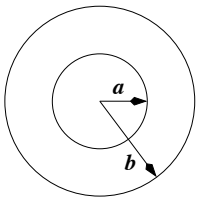
Suppose a spherical capacitor has a permittivity of $2.1\varepsilon_0$ (which is the permittivity of Teflon®) and the electric field is given by

$$\mathbf{E} = \frac{2}{r^2}\hat{\mathbf{a}}_r \ \text{mV/m} \tag{1}$$

How much energy is stored in the capacitor if $a = 10$ cm and $b = 20$ cm?



The equation for the energy stored in a volume of space is given by

$$W_E = \frac{1}{2}\int_v \varepsilon E^2 dv \ \text{[J]} \tag{2}$$

Because we're considering the energy stored in a spherical capacitor, we use spherical coordinates. Using **E** from Eq. (1) and dv from the yellow sheet, Eq. (2) becomes

$$\begin{aligned}
W_E &= \frac{1}{2}(2.1\varepsilon_0)\int_v \left(\frac{4\times 10^{-6}}{r^4}\right) r^2 \sin\theta dr d\theta d\phi \\
&= 4.2\varepsilon_0 \times 10^{-6}\int_{0.1}^{0.2}\frac{1}{r^2}dr\int_0^{\pi}\sin\theta d\theta\int_0^{2\pi} d\phi \\
&= 4.2\varepsilon_0 \times 10^{-6}\left(\frac{1}{r}\,|_{0.2}^{0.1}\right)(\cos\theta\,|_{\pi}^{0})(\phi\,|_0^{2\pi}) \\
&= 4.2\varepsilon_0 \times 10^{-6}\left(\frac{1}{0.1}-\frac{1}{0.2}\right)(2)(2\pi) \\
&\approx 2.3365 \ \text{fJ}
\end{aligned}$$