EE331—EXAMPLE #21: RELAXATION TIME

At $t = 0$ an excess charge of 500 trillion (5×10^{14}) electrons is dumped inside a lossy sphere of material of radius 4 cm. (a) What is the charge density at $t = 0$? (Recall that $Q = -1.6 \times 10^{-19}$ C.) (b) If the dielectric constant for the lossy material is 1 and its resistivity is $2.83 \times 10^{-8}\Omega$·m, what is the relaxation time? (c) Is the material more likely a conductor or a lossy dielectric? (d) How long does it take for the charge to decrease to 80% of its initial value?

(a) At $t = 0$:

$$\rho_{v0} = \frac{NQ}{v} = \frac{NQ}{\frac{4}{3}\pi r^3} = \frac{(5 \times 10^{14})(-1.6 \times 10^{-19})}{\frac{4}{3}\pi(0.04)^3} = -0.298416 \ \ [\mathrm{C/m^3}]$$

(b) For $\varepsilon_r = 1$ and $\rho_c = 2.83 \times 10^{-8} \ \ [\Omega \cdot \mathrm{m}]$:

$$T_r = \frac{\varepsilon}{\sigma} = \varepsilon_r \varepsilon_0 \rho_c = (1)(8.854 \times 10^{-12})(2.83 \times 10^{-8}) = 2.505682 \times 10^{-19} \ \ [\mathrm{s}]$$

(c) Because the relaxation time is so short, it's probably a conductor.

(d) Recall that the equation for the volume charge density is given by:

$$\rho_v(t) = \rho_{v0} e^{-t/T_r}$$

We want $\rho_v(t) = 0.8\rho_{v_0}$, so:

$$0.8\rho_{v_0} = \rho_{v_0} e^{-t/T_r}$$

Taking the natural logarithm of both sides:

$$\ln 0.8 = \frac{-t}{T_r}$$

Now solve for t:

$$\begin{aligned} t &= (-T_r)(\ln 0.8) \\ &= (-2.505682 \times 10^{-19})(-0.223144) \\ &= 5.591268 \times 10^{-20} \text{ s} \end{aligned}$$