EE331—EXAMPLE #22: BOUNDARY CONDITION I

Region 2 ($z < 0$) contains a dielectric for which $\varepsilon_r = 2.5$. Region 1 ($z > 0$) has $\varepsilon_r = 4$. If $\mathbf{E}_2 = -30\widehat{\mathbf{a}}_x + 50\widehat{\mathbf{a}}_y + 70\widehat{\mathbf{a}}_z$ V/m, find (a) $\mathbf{D}_1$ and (b) the angle between $\mathbf{E}_2$ and the normal to the surface.

First make a sketch:

1 $\quad \varepsilon_{r_1} = 4$, $\mathbf{E}_1$, $\mathbf{D}_1$

———————————————— $z = 0$

2 $\quad \varepsilon_{r_2} = 2.5$, $\mathbf{E}_2$, $\mathbf{D}_2$

(a) Write down the dielectric-dielectric boundary conditions. Also, recall that $\mathbf{D} = \varepsilon\mathbf{E}$.

BC:
1) $E_{1_t} = E_{2_t}$
2) $D_{1_n} = D_{2_n}$ since no ρ_S specified

Next identify the components of $\mathbf{E}_2$ that are tangential and normal to the surface:

$$\mathbf{E}_{2_t} = -30\widehat{\mathbf{a}}_x + 50\widehat{\mathbf{a}}_y$$

$$\mathbf{E}_{2_n} = 70\widehat{\mathbf{a}}_z$$

$$\rightarrow\ \mathbf{D}_{2_n} = \varepsilon_2\mathbf{E}_{2_n} = \varepsilon_{r2}\varepsilon_0\mathbf{E}_{2_n} = 2.5\varepsilon_0(70\widehat{\mathbf{a}}_z) = 175\varepsilon_0\widehat{\mathbf{a}}_z$$

Apply the BC to get:

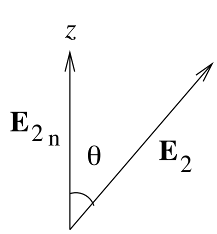
$$\mathbf{E}_{2_t} = -30\widehat{\mathbf{a}}_x + 50\widehat{\mathbf{a}}_y = \mathbf{E}_{1_t}$$

$$\rightarrow\ \mathbf{D}_{1_t} = \varepsilon_1\mathbf{E}_{1_t} = 4\varepsilon_0(-30\widehat{\mathbf{a}}_x + 50\widehat{\mathbf{a}}_y) = -120\varepsilon_0\widehat{\mathbf{a}}_x + 200\varepsilon_0\widehat{\mathbf{a}}_y$$

$$\mathbf{D}_{1_n} = \mathbf{D}_{2_n} = 175\varepsilon_0\widehat{\mathbf{a}}_z$$

$$\Rightarrow \mathbf{D}_1 = \mathbf{D}_{1_t} + \mathbf{D}_{1_n} = -120\varepsilon_0\widehat{\mathbf{a}}_x + 200\varepsilon_0\widehat{\mathbf{a}}_y + 175\varepsilon_0\widehat{\mathbf{a}}_z \text{ C/m}^2$$

(b) We want to find the angle between $\mathbf{E}_2$ and the normal to the surface which is the same thing as finding the angle between $\mathbf{E}_2$ and $\mathbf{E}_{2_n}$.



$$E_{2_n} = E_2\cos\theta$$

$$\rightarrow\ \cos\theta = \frac{E_{2_n}}{E_2}$$

$$\rightarrow\ \theta = \cos^{-1}\left(\frac{E_{2_n}}{E_2}\right) = \cos^{-1}\left(\frac{70}{\sqrt{30^2+50^2+70^2}}\right)$$

$$\Rightarrow\ \theta = 39.79^\circ$$