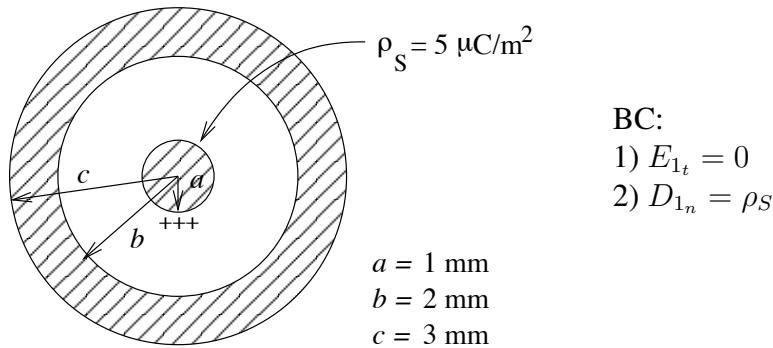


EE331—EXAMPLE #23: BOUNDARY CONDITION II

A coaxial cable has an inner conductor of radius 1 mm and an outer conductor with inner radius of 3 mm and outer radius of 4 mm. Teflon, with a dielectric constant of 2.1, fills the region between the two conductors. The surface charge density on the inner conductor is $5 \mu\text{C}/\text{m}^2$. (a) What is the electric flux density \mathbf{D} in the Teflon immediately adjacent to (next to) the inner conductor? (b) What is \mathbf{D} in the Teflon immediately adjacent to the inner radius of the outer conductor?

- (a) Make a sketch and identify knowns. Since we're dealing with a dielectric next to a conductor, we write down the two dielectric-conductor boundary conditions.



We're asked for the electric flux density in the dielectric right next to the inner conductor, so we use the second boundary condition:

$$D_{1n} = D_{\rho_1} = 5 \mu\text{C}/\text{m}^2 \rightarrow \mathbf{D} = 5\hat{\mathbf{a}}_\rho \mu\text{C}/\text{m}^2$$

Note that we choose the unit vector $\hat{\mathbf{a}}_\rho$ to be positive since the electric flux density points away from the positive charge.

- (b) This part also uses the second boundary condition, but what is ρ_S ? It's not $5 \mu\text{C}/\text{m}^2$. Why not? Because the total charge on the outer conductor must be equal in magnitude and opposite in sign to the charge on the inner conductor. However, since the surface area of the outer conductor is larger than that of the inner conductor, the surface charge density isn't the same.

$$Q_{in} = \rho_{S_{in}} S_{in} = (5 \times 10^{-6})(2\pi\rho_{in}L) = \pi \times 10^{-8} L \text{ C}$$

$$D_{1n} = \rho_{S_{out}} = \frac{Q_{out}}{S_{out}} = \frac{-Q_{in}}{S_{out}} = \frac{-\pi \times 10^{-8} L}{2\pi\rho_{out}L} = \frac{-10^{-8}}{6 \times 10^{-3}} = -1.67 \mu\text{C}/\text{m}^2$$

where S_{in} and S_{out} are the surface areas of the inner conductor and the inner surface of the outer conductor, respectively. Finally,

$$\mathbf{D} = 1.67\hat{\mathbf{a}}_\rho \mu\text{C}/\text{m}^2$$

Again \mathbf{D} points in the positive $\hat{\mathbf{a}}_\rho$ direction.