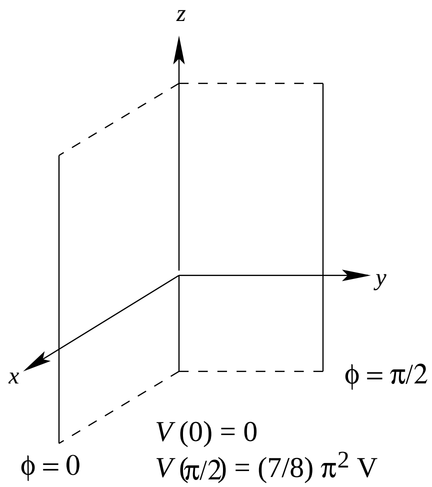
Two conducting sheets are placed along the $x$ and $y$ axes in free space so that their edges meet, but don't touch, along the $z$ axis. The sheet along the $x$ axis is grounded, and the sheet along the $y$ axis is held at $7\pi^2/8$ V. A volume charge density of $\rho_v = \varepsilon_0/\rho^2$ C/m$^3$ exists between the two sheets. Assuming that the sheets extend to $\pm\infty$ in the $z$ direction, find the potential between the conducting sheets.




First, since a volume charge density is given, we need to use Poisson's equation. We write:

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon} = -\frac{1}{\rho^2}$$

Since we're looking for the potential in the region between the plates, we use cylindrical coordinates. Because the potential is only a function of $\phi$ and not of $\rho$ or $z$ (why?), Poisson's equation becomes:

$$\frac{1}{\rho^2}\frac{d^2V}{d\phi^2} = -\frac{1}{\rho^2}$$

Multiplying both sides by $\rho^2$ gives:

$$\frac{d^2V}{d\phi^2} = -1$$

Next we integrate twice wrt $\phi$:

$$\frac{dV}{d\phi} = -\phi + A$$

$$V(\phi) = -\frac{1}{2}\phi^2 + A\phi + B$$

To find the constants $A$ and $B$, we apply the values of $V$ at the boundary. Note that you have to use radians, not degrees!

$$V(0) = 0 + 0 + B = 0 \quad \rightarrow \quad B = 0$$

$$V(\pi/2) = -\frac{1}{2}\left(\frac{\pi}{2}\right)^2 + A\left(\frac{\pi}{2}\right) = \frac{7}{8}\pi^2 \quad \rightarrow \quad A = 2\pi$$

Thus, our final answer is given by:

$$V(\phi) = -\frac{1}{2}\phi^2 + 2\pi\phi \quad \text{V}$$

Check to make sure your answer makes sense! Also, check to make sure the values at the boundaries are satisfied.