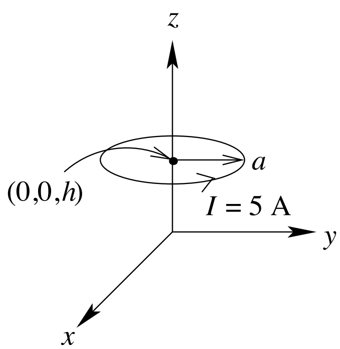
Current of 5 A flows in the positive $\phi$ direction in a circular ring of radius = $a$ m. The ring is located parallel to the $z = 0$ plane and is centered at $(0, 0, h)$ m. Find $\mathbf{H}$ along the $z$ axis.




First make a sketch and choose the appropriate coordinate system. Next write down the appropriate form of the BS law. In this case we have a line current so:

$$\mathbf{H}(\mathbf{r}) = \frac{1}{4\pi} \int \frac{I\mathbf{dl}' \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} \qquad \text{[A/m]}$$

Next we identify all the parameters:

$$\mathbf{r} = z\,\widehat{\mathbf{a}}_z \quad \text{[m]}$$

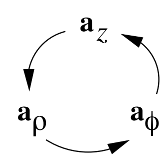
$$\mathbf{r}' = a\,\widehat{\mathbf{a}}_\rho + h\,\widehat{\mathbf{a}}_z \quad \text{[m]}$$

$$\mathbf{r} - \mathbf{r}' = -a\,\widehat{\mathbf{a}}_\rho + (z-h)\,\widehat{\mathbf{a}}_z \quad \text{[m]}$$

$$\mathbf{dl}' = a\,d\phi'\,\widehat{\mathbf{a}}_\phi \quad \text{[m]}$$

$$|\mathbf{r} - \mathbf{r}'|^3 = [(z-h)^2 + a^2]^{3/2} \quad \text{[m}^3\text{]}$$

$$\mathbf{dl}' \times (\mathbf{r} - \mathbf{r}') = a\,d\phi'\,\widehat{\mathbf{a}}_\phi \times (-a\,\widehat{\mathbf{a}}_\rho + (z-h)\,\widehat{\mathbf{a}}_z)) = a^2\,d\phi'\,\widehat{\mathbf{a}}_z + a(z-h)\,d\phi'\,\widehat{\mathbf{a}}_\rho$$




and the limits of integration are $0 \to 2\pi$. Thus,

$$\begin{aligned}
\mathbf{H}(z) &= \frac{5}{4\pi} \int_0^{2\pi} \frac{a^2\,d\phi'\,\widehat{\mathbf{a}}_z + \cancelto{0}{a(z-h)\,d\phi'\,\widehat{\mathbf{a}}_\rho}}{[(z-h)^2 + a^2]^{3/2}} \\
&= \frac{5a^2}{4\pi} \frac{1}{[(z-h)^2 + a^2]^{3/2}}\,\widehat{\mathbf{a}}_z \int_0^{2\pi} d\phi' \\
&= \frac{2.5a^2}{[(z-h)^2 + a^2]^{3/2}}\,\widehat{\mathbf{a}}_z \quad \text{[A/m]}
\end{aligned}$$

Note: Always choose $\mathbf{dl}'$ to be positive, and choose the limits of integration according to the direction of the current relative to the positive direction:

$$\int_0^{2\pi} d\phi' \qquad\qquad \int_{2\pi}^{0} d\phi'$$