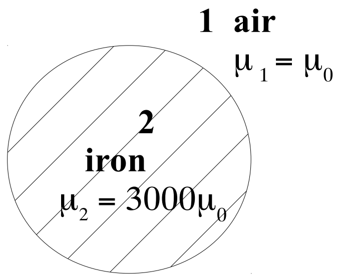
An iron cylinder surrounded by air has a relative permeability of 3000. The magnetic field adjacent to the cylinder in air is found to be $\mathbf{H}_1 = 3\hat{\mathbf{a}}_\rho + 2\hat{\mathbf{a}}_\phi + 4\hat{\mathbf{a}}_z$ A/m. (a) Find the magnetic flux density in the air adjacent to the cylinder. (b) Find the magnetic flux density inside the iron cylinder.

Make a sketch and write down the boundary conditions. Because we're dealing with magnetic fields, the BC are the same for both dielectric-dielectric and dielectric-conductor boundaries. As no current is given, we write down the two BC without any source current. We also write down the relationship between **H** and **B**.



BC:
1) $H_{1t} = H_{2t}$
2) $B_{1n} = B_{2n}$

$\mathbf{B} = \mu\mathbf{H}$

(a) This part doesn't require use of the BC. We just need to find $\mathbf{B}_1$ given $\mathbf{H}_1$ in air:

$$\begin{aligned}\mathbf{B}_1 = \mu_0\mathbf{H}_1 &= (4\pi \times 10^{-7})(3\hat{\mathbf{a}}_\rho + 2\hat{\mathbf{a}}_\phi + 4\hat{\mathbf{a}}_z) \\ &= 3.7699\hat{\mathbf{a}}_\rho + 2.5133\hat{\mathbf{a}}_\phi + 5.0265\hat{\mathbf{a}}_z \ \ \mu\text{Wb/m}^2 \qquad (1)\end{aligned}$$

(b) Next we want to find $\mathbf{B}_2$ in the iron so we need to identify the normal and tangential components of the problem for use in the BC. Because we're considering a cylinder, the normal component must be in the $\hat{\mathbf{a}}_\rho$ direction. We use the second BC together with the result from Eq. (1) to get:

$$\mathbf{B}_{2n} = \mathbf{B}_{1n} = 3.7699\hat{\mathbf{a}}_\rho \ \ \mu\text{Wb/m}^2 \qquad (2)$$

The tangential components are in the $\hat{\mathbf{a}}_\phi$ and $\hat{\mathbf{a}}_z$ directions for a cylinder. We use the first BC together with the information given in the problem to get:

$$\mathbf{H}_{2t} = \mathbf{H}_{1t} = 2\hat{\mathbf{a}}_\phi + 4\hat{\mathbf{a}}_z \ \ \text{A/m}$$

We use the relationship between **H** and **B** to find the tangential component of the magnetic flux density:

$$\begin{aligned}\mathbf{B}_{2t} = \mu\mathbf{H}_{2t} &= 3000\mu_0(2\hat{\mathbf{a}}_\phi + 4\hat{\mathbf{a}}_z) \\ &= 7.5398\hat{\mathbf{a}}_\phi + 15.0796\hat{\mathbf{a}}_z \ \ \text{mWb/m}^2 \qquad (3)\end{aligned}$$

Finally, combining the results of Eqs. (2) and (3) gives:

$$\mathbf{B}_2 = (3.7699 \times 10^{-3})\hat{\mathbf{a}}_\rho + 7.5398\hat{\mathbf{a}}_\phi + 15.0796\hat{\mathbf{a}}_z \ \ \text{mWb/m}^2$$

The effect of the permeability of iron is to make the magnitude of the magnetic flux density almost 2500 times larger than in air.