

### EE331—EXAMPLE #3: REFLECTION COEFFICIENT

Suppose we have a lossless line with a characteristic impedance of  $Z_0$   $[\Omega]$ . (a) What is the reflection coefficient for a short-circuited line? (b) What is  $\Gamma_L$  for an open-circuited line? (c) What is the maximum magnitude of  $\Gamma_L$ ? (d) What is the minimum magnitude of the reflection coefficient? (e) What value of  $Z_0$  gives maximum power transfer to the load? (f) What is the reflected voltage when total power is delivered to the load?

First we write down the equation for the reflection coefficient:

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{V_0^-}{V_0^+} = |\Gamma_L|e^{j\theta_r}$$

(a) For a short-circuited line,  $Z_L = 0$ . Thus,

$$\Gamma_L = \frac{0 - Z_0}{0 + Z_0} = -1$$

(b) For an open-circuited line,  $Z_L \rightarrow \infty$ . Thus,

$$\Gamma_L = \frac{1 - Z_0/Z_L}{1 + Z_0/Z_L} = \frac{1}{1} = 1$$

(c) Extreme values of  $Z_L$  are 0 and  $\infty$ . Thus  $|\Gamma_L|_{\max} = 1$ .

(d) When  $Z_L = Z_0$ ,  $\Gamma_L = 0$ . Thus,  $|\Gamma_L|_{\min} = 0$ .

(e) Maximum power transfer occurs when there is no reflection at the load—that is, when  $\Gamma_L = 0$ , so from (d),  $Z_L = Z_0$ . When this occurs, we say the load is matched to the line.

(f) Since  $V_0^- = \Gamma_L V_0^+$  and  $\Gamma_L = 0$ ,  $V_0^- = 0$ —that is, no voltage is reflected.