EE331—EXAMPLE #4: CONVERSION OF $V_s(z)$ TO $V(z,t)$

The general solution for the voltage on a transmission line in the phasor world is given by:

$$V_s(z) = V_0^+ \left(e^{-\gamma z} + \Gamma_L e^{\gamma z} \right) \quad [\text{V}]$$

Convert this to the time domain.

$$\begin{aligned} V(z,t) &= \text{Re}\left[V_s(z) e^{j\omega t} \right] \\ &= \text{Re}\left[V_0^+ \left(e^{-\gamma z} + \Gamma_L e^{\gamma z} \right) e^{j\omega t} \right] \\ &= \text{Re}\left[|V_0^+| e^{j\phi} \left(e^{-\alpha z - j\beta z} + |\Gamma_L| e^{j\theta_r} e^{\alpha z + j\beta z} \right) e^{j\omega t} \right] \\ &= \text{Re}\left[|V_0^+| \left(e^{-\alpha z} e^{j(\omega t - \beta z + \phi)} + |\Gamma_L| e^{\alpha z} e^{j(\omega t + \beta z + \theta_r + \phi)} \right) \right] \\ &= |V_0^+| \left[e^{-\alpha z} \cos(\omega t - \beta z + \phi) + |\Gamma_L| e^{\alpha z} \cos(\omega t + \beta z + \theta_r + \phi) \right] \quad [\text{V}] \end{aligned}$$

where

$\gamma = \alpha + j\beta$ = propagation constant [1/m]
α = attenuation constant [Np/m]
β = phase constant [rad/m]
ω = radian frequency of operation [rad/s]
$|V_0^+|$ = magnitude of V_0^+ [V]
ϕ = phase angle of V_0^+ [rad]
$|\Gamma_L|$ = magnitude of the reflection coefficient [.]
θ_r = phase angle of the reflection coefficient [rad]