The input impedance of a line Z_{in} can be generalized to be the "input" impedance of a line looking toward the load measured anywhere along the line.

$$Z'_{\text{in}} = \frac{V_s(z)}{I_s(z)} = Z_0 \left(\frac{Z_L - Z_0 \tanh(\gamma z)}{Z_0 - Z_L \tanh(\gamma z)} \right) \tag{1}$$

where z is a negative value (since the load is at $z = 0$). For a lossless line this becomes:

$$Z'_{\text{in}} = \frac{V_s(z)}{I_s(z)} = Z_0 \left(\frac{Z_L - jZ_0 \tan(\beta z)}{Z_0 - jZ_L \tan(\beta z)} \right) \tag{2}$$

(Why?) For a lossless line of length l m, the input impedance at the generator is:

$$Z_{\text{in}} = \frac{V_s(-l)}{I_s(-l)} = Z_0 \left(\frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)} \right) \tag{3}$$

(a) Why is there a sign change between Eqs. (2) and (3)? (b) Something magical happens to the input impedance when l is a multiple of $\lambda/2$—i.e., a multiple of half a wavelength. What does it become?

(a) There's a sign change because $\tan(\beta z)$ is an odd function:

$$\tan(\beta z) = \frac{\sin(\beta z)}{\cos(\beta z)} = \frac{\sin(-\beta l)}{\cos(-\beta l)} = \frac{-\sin(\beta l)}{\cos(\beta l)} = -\tan(\beta l)$$

(b) For $l = \dfrac{n\lambda}{2}$, $n = 1,\ 2,\ 3, \ldots$

$$\beta l = \frac{2\pi}{\lambda} \cdot \frac{n\lambda}{2} = n\pi$$

and so

$$\tan(\beta l) = \tan(n\pi) = \frac{\sin(n\pi)}{\cos(n\pi)} = 0$$

Thus,

$$Z_{in} = Z_0 \left(\frac{Z_L + 0}{Z_0 + 0} \right)$$

or

$$Z_{in} = Z_L$$

for $l =$ multiples of $\lambda/2$! In words, when the length of a transmission line is a multiple of half a wavelength, the input impedance is equal to the load impedance. Cool, eh?