EE331—Example #7: TL Average Power to the Load

A transmission line with a characteristic impedance of 50 Ω is connected to a load with an impedance of 100 Ω. What fraction of power is delivered to this load relative to the power delivered if the load were matched?

The average power delivered to the load is given by:

$$P_{L_{ave}} = \frac{1}{2}\frac{|V_0^+|^2}{Z_0}\left(1 - |\Gamma_L|^2\right) \quad [\mathrm{W}]$$

For a perfectly-matched load, $|\Gamma_L| = 0$ (i.e., there's no reflection). Thus,

$$P_{L_{max}} = \frac{1}{2}\frac{|V_0^+|^2}{Z_0} \quad [\mathrm{W}]$$

For our problem, $Z_0 = 50\ \Omega$ and $Z_L = 100\ \Omega$. Therefore, Γ_L is:

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100 - 50}{100 + 50} = \frac{50}{150} = \frac{1}{3}$$

and

$$|\Gamma_L| = \frac{1}{3}$$

The fraction of power is given by $P_{L_{ave}}/P_{L_{max}}$:

$$P_{L_{ave}}/P_{L_{max}} = \frac{\frac{1}{2}\frac{|V_0^+|^2}{Z_0}(1 - \frac{1}{3}^2)}{\frac{1}{2}\frac{|V_0^+|^2}{Z_0}} = 1 - \frac{1}{9} = \frac{8}{9}$$