For a lossless line with $Z_0 = 60 \Omega$, the maximum Z_{in} is 180 Ω and occurs at $\lambda/24 \approx 0.042\lambda$ from the load. If the line is 0.3λ long, find (a) s, (b) Z_L , (c) Z_{in} , and (d) the location of the first voltage minimum.

(a) $Z_{in_{max}}$ occurs at $|V_s|_{max}$ (and $|I_s|_{min}$). On the Smith chart, voltage maxima are located on the positive Γ_r axis and voltage minima are located on the negative Γ_r axis (recall that maxima and minima are $\lambda/4$ apart). Normalize $Z_{in_{max}}$:

$$z_{in_{max}} = \frac{Z_{in_{max}}}{Z_0} = \frac{180}{60} = 3.$$

Mark $z_{in_{max}} = 3 + j0$ on the Smith chart, and draw the SWR circle through it. By inspection:

$$s = 3$$

(b) The maximum Z_{in} occurs 0.042λ from the load. Thus, to find Z_L move 0.042λ toward the load and draw a line from the origin through this point. This is z_L :

$$z_L = 1.96 + j1.30$$

Thus,

$$Z_L = z_L Z_0 = (1.96 + j1.30)(60) = 117.6 + j78 \,\Omega$$

(c) The line is 0.3λ long so move from the load point toward the generator 0.3λ , and mark the location. Be careful with the scales. Again draw a line from the origin through the new location. The intersection with the SWR circle gives the input impedance:

$$z_{in} = 0.34 + j0.05$$

Thus,

$$Z_{in} = z_{in} Z_0 = (0.34 + j0.05)(60) = 20.4 + j3 \Omega$$

(d) $|V_s|_{min}$ is located $\lambda/4$ from $|V_s|_{max}$. Since $|V_s|_{max}$ is at 0.042 λ :

$$l_{min} = 0.042\lambda + 0.25\lambda = 0.292\lambda$$
 from the load

If the value of λ were given, the actual distance could be calculated.

Note that fractions of λ and degrees can be used interchangeably. For example, $\lambda/24 \rightarrow 30^{\circ}$ since $(\lambda/24)(720^{\circ}/\lambda) = 30^{\circ}$.

The Complete Smith Chart

Black Magic Design

