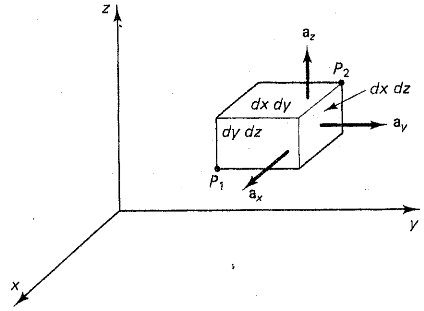
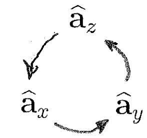
EE 331 — Coordinate Systems

Cartesian (x, y, z)

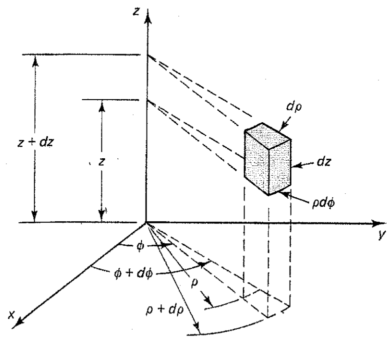
$$d\bar{\mathbf{l}} = dx\,\widehat{\mathbf{a}}_x + dy\,\widehat{\mathbf{a}}_y + dz\,\widehat{\mathbf{a}}_z \text{ [m]}$$
$$d\bar{\mathbf{S}} = \pm dy\,dz\,\widehat{\mathbf{a}}_x \text{ [m}^2\text{]}$$
$$d\bar{\mathbf{S}} = \pm dx\,dz\,\widehat{\mathbf{a}}_y \text{ [m}^2\text{]}$$
$$d\bar{\mathbf{S}} = \pm dx\,dy\,\widehat{\mathbf{a}}_z \text{ [m}^2\text{]}$$
$$dv = dx\,dy\,dz \text{ [m}^3\text{]}$$
$$\bar{\mathbf{r}} = x\,\widehat{\mathbf{a}}_x + y\,\widehat{\mathbf{a}}_y + z\,\widehat{\mathbf{a}}_z \text{ [m]}$$

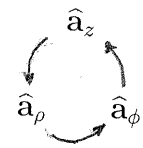




Cylindrical (ρ, ϕ, z)

$$d\bar{\mathbf{l}} = d\rho\,\widehat{\mathbf{a}}_\rho + \rho\,d\phi\,\widehat{\mathbf{a}}_\phi + dz\,\widehat{\mathbf{a}}_z \text{ [m]}$$
$$d\bar{\mathbf{S}} = \pm \rho\,d\phi\,dz\,\widehat{\mathbf{a}}_\rho \text{ [m}^2\text{]}$$
$$d\bar{\mathbf{S}} = \pm d\rho\,dz\,\widehat{\mathbf{a}}_\phi \text{ [m}^2\text{]}$$
$$d\bar{\mathbf{S}} = \pm \rho\,d\rho\,d\phi\,\widehat{\mathbf{a}}_z \text{ [m}^2\text{]}$$
$$dv = \rho\,d\rho\,d\phi\,dz \text{ [m}^3\text{]}$$
$$\bar{\mathbf{r}} = \rho\,\widehat{\mathbf{a}}_\rho + z\,\widehat{\mathbf{a}}_z \text{ [m]}$$





Cartesian → Cylindrical

$$A_\rho = \bar{\mathbf{A}}\cdot\widehat{\mathbf{a}}_\rho = A_x\,\widehat{\mathbf{a}}_x\cdot\widehat{\mathbf{a}}_\rho + A_y\,\widehat{\mathbf{a}}_y\cdot\widehat{\mathbf{a}}_\rho$$
$$A_\phi = \bar{\mathbf{A}}\cdot\widehat{\mathbf{a}}_\phi = A_x\,\widehat{\mathbf{a}}_x\cdot\widehat{\mathbf{a}}_\phi + A_y\,\widehat{\mathbf{a}}_y\cdot\widehat{\mathbf{a}}_\phi$$
$$A_z = \bar{\mathbf{A}}\cdot\widehat{\mathbf{a}}_z = A_z$$

- $\bar{\mathbf{A}} = A_\rho\,\widehat{\mathbf{a}}_\rho + A_\phi\,\widehat{\mathbf{a}}_\phi + A_z\widehat{\mathbf{a}}_z$

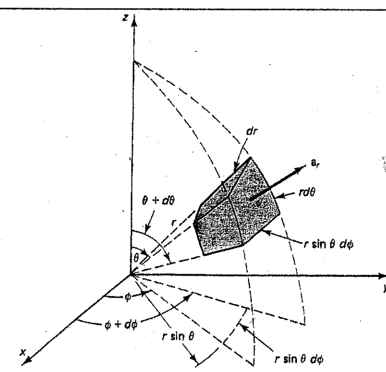
Cylindrical → Cartesian

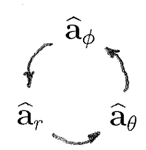
$$A_x = \bar{\mathbf{A}}\cdot\widehat{\mathbf{a}}_x = A_\rho\,\widehat{\mathbf{a}}_\rho\cdot\widehat{\mathbf{a}}_x + A_\phi\,\widehat{\mathbf{a}}_\phi\cdot\widehat{\mathbf{a}}_x$$
$$A_y = \bar{\mathbf{A}}\cdot\widehat{\mathbf{a}}_y = A_\rho\,\widehat{\mathbf{a}}_\rho\cdot\widehat{\mathbf{a}}_y + A_\phi\,\widehat{\mathbf{a}}_\phi\cdot\widehat{\mathbf{a}}_y$$
$$A_z = \bar{\mathbf{A}}\cdot\widehat{\mathbf{a}}_z = A_z$$

- $\bar{\mathbf{A}} = A_x\,\widehat{\mathbf{a}}_x + A_y\,\widehat{\mathbf{a}}_y + A_z\widehat{\mathbf{a}}_z$

Spherical (r, θ, ϕ)

$$d\bar{\mathbf{l}} = dr\,\widehat{\mathbf{a}}_r + r\,d\theta\,\widehat{\mathbf{a}}_\theta + r\,\sin\theta\,d\phi\,\widehat{\mathbf{a}}_\phi \text{ [m]}$$
$$d\bar{\mathbf{S}} = \pm r^2\sin\theta\,d\theta\,d\phi\,\widehat{\mathbf{a}}_r \text{ [m}^2\text{]}$$
$$d\bar{\mathbf{S}} = \pm r\,\sin\theta\,dr\,d\phi\,\widehat{\mathbf{a}}_\theta \text{ [m}^2\text{]}$$
$$d\bar{\mathbf{S}} = \pm r\,dr\,d\theta\,\widehat{\mathbf{a}}_\phi \text{ [m}^2\text{]}$$
$$dv = r^2\sin\theta\,dr\,d\theta\,d\phi \text{ [m}^3\text{]}$$
$$\bar{\mathbf{r}} = r\,\widehat{\mathbf{a}}_r \text{ [m]}$$





Cartesian → Spherical

$$A_r = \bar{\mathbf{A}}\cdot\widehat{\mathbf{a}}_r = A_x\,\widehat{\mathbf{a}}_x\cdot\widehat{\mathbf{a}}_r + A_y\,\widehat{\mathbf{a}}_y\cdot\widehat{\mathbf{a}}_r + A_z\,\widehat{\mathbf{a}}_z\cdot\widehat{\mathbf{a}}_r$$
$$A_\theta = \bar{\mathbf{A}}\cdot\widehat{\mathbf{a}}_\theta = A_x\,\widehat{\mathbf{a}}_x\cdot\widehat{\mathbf{a}}_\theta + A_y\,\widehat{\mathbf{a}}_y\cdot\widehat{\mathbf{a}}_\theta + A_z\widehat{\mathbf{a}}_z\cdot\widehat{\mathbf{a}}_\theta$$
$$A_\phi = \bar{\mathbf{A}}\cdot\widehat{\mathbf{a}}_\phi = A_x\,\widehat{\mathbf{a}}_x\cdot\widehat{\mathbf{a}}_\phi + A_y\,\widehat{\mathbf{a}}_y\cdot\widehat{\mathbf{a}}_\phi$$

- $\bar{\mathbf{A}} = A_r\,\widehat{\mathbf{a}}_r + A_\theta\,\widehat{\mathbf{a}}_\theta + A_\phi\,\widehat{\mathbf{a}}_\phi$

Spherical → Cartesian

$$A_x = \bar{\mathbf{A}}\cdot\widehat{\mathbf{a}}_x = A_r\,\widehat{\mathbf{a}}_r\cdot\widehat{\mathbf{a}}_x + A_\theta\,\widehat{\mathbf{a}}_\theta\cdot\widehat{\mathbf{a}}_x + A_\phi\,\widehat{\mathbf{a}}_\phi\cdot\widehat{\mathbf{a}}_x$$
$$A_y = \bar{\mathbf{A}}\cdot\widehat{\mathbf{a}}_y = A_r\,\widehat{\mathbf{a}}_r\cdot\widehat{\mathbf{a}}_y + A_\theta\,\widehat{\mathbf{a}}_\theta\cdot\widehat{\mathbf{a}}_y + A_\phi\,\widehat{\mathbf{a}}_\phi\cdot\widehat{\mathbf{a}}_y$$
$$A_z = \bar{\mathbf{A}}\cdot\widehat{\mathbf{a}}_z = A_r\,\widehat{\mathbf{a}}_r\cdot\widehat{\mathbf{a}}_z + A_\theta\,\widehat{\mathbf{a}}_\theta\cdot\widehat{\mathbf{a}}_z$$

- $\bar{\mathbf{A}} = A_x\,\widehat{\mathbf{a}}_x + A_y\,\widehat{\mathbf{a}}_y + A_z\widehat{\mathbf{a}}_z$

Conversion Factors

Cartesian/Cylindrical

$x = \rho\,\cos\phi$ | $\rho = \sqrt{x^2 + y^2}$
$y = \rho\,\sin\phi$ | $\phi = \tan^{-1}(y/x)$
$z = z$ | $z = z$

Cartesian/Spherical

$x = r\,\sin\theta\,\cos\phi$ | $r = \sqrt{x^2 + y^2 + z^2}$
$y = r\,\sin\theta\,\sin\phi$ | $\theta = \cos^{-1}(z/r)$
$z = r\,\cos\theta$ | $\phi = \tan^{-1}(y/x)$

$\cdot$	$\widehat{\mathbf{a}}_\rho$	$\widehat{\mathbf{a}}_\phi$	$\widehat{\mathbf{a}}_z$
$\widehat{\mathbf{a}}_x$	$\cos\phi$	$-\sin\phi$	0
$\widehat{\mathbf{a}}_y$	$\sin\phi$	$\cos\phi$	0
$\widehat{\mathbf{a}}_z$	0	0	1

$\cdot$	$\widehat{\mathbf{a}}_r$	$\widehat{\mathbf{a}}_\theta$	$\widehat{\mathbf{a}}_\phi$
$\widehat{\mathbf{a}}_x$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\widehat{\mathbf{a}}_y$	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\widehat{\mathbf{a}}_z$	$\cos\theta$	$-\sin\theta$	0