Name _____

EE 331 – Exam #1
Feb. 14, 2020
Closed Book/Closed Notes

Relax and think carefully. Show all your work to insure maximum partial credit. Remember that *neatness counts*! **SHOW ALL UNITS (e.g., V, Ω, m), and COMMUNICATE WHAT YOU KNOW!!**

1(a). We manage to design a transmission line with the following TL parameters for an operating frequency of $f = \frac{1}{2\pi} \times 10^8$ Hz:

$$R = 0, \quad L = 20 \text{ nH/m}, \quad G = 0, \quad C = 2 \text{ pF/m}$$

Find:

(i) (4 pts) $\alpha$

$$\alpha = 0 \ \ (R = 0, G = 0 \text{ so TL is lossless})$$

(ii) (5 pts) $\beta$

$$\begin{aligned} \beta &= \omega\sqrt{LC} = 2\pi f\sqrt{LC} = (2\pi)\left(\frac{10^8}{2\pi}\right)\sqrt{(20\times 10^{-9})(2\times 10^{-12})} \\ &= 10^8\sqrt{4\times 10^{-20}} = (10^8)(2\times 10^{-10}) \\ &= 2\times 10^{-2} = 0.02 \text{ rad/m} \end{aligned}$$

(iii) (5 pts) $Z_0$

$$\begin{aligned} Z_0 &= \sqrt{L/C} = \sqrt{(20\times 10^{-9})/(2\times 10^{-12})} \\ &= \sqrt{10^4} = 10^2 = 100\ \Omega \end{aligned}$$

1(b). A 50-cm long, 50-Ω lossless line is used as a quarter-wave transformer to match a resistive load to a 100-Ω line. Assuming $u = c = 3 \times 10^8$ m/s, find:

(i) (5 pts) the operating frequency $f$ of the line

$$\begin{aligned} f &= u/\lambda = \frac{u}{(4)(\lambda/4)} = \frac{3\times 10^8}{(4)(0.5)} \\ &= \frac{3\times 10^8}{2} = 1.5\times 10^8 = 150 \text{ MHz} \end{aligned}$$

(ii) (6 pts) the value of the load impedance matched to the line

$$\begin{aligned} Z'_{in} &= \frac{Z_0'^2}{Z_L} \rightarrow Z_L = \frac{Z_0'^2}{Z'_{in}} \\ Z_L &= \frac{(50)^2}{100} = \frac{2500}{100} = 25\ \Omega \end{aligned}$$

2. A lossless line has a characteristic impedance of 50 $\Omega$. A load is connected to the line. The reflection coefficient is measured to be $\Gamma_{_L} = 0.2$ (i.e., it's real).

   (a) (5 pts) Find the standing wave ratio.

$$\begin{aligned} s &= \frac{1+|\Gamma_L|}{1-|\Gamma_L|} \\ &= \frac{1+0.2}{1-0.2} = \frac{1.2}{0.8} = 1.5 \end{aligned}$$

   (b) (6 pts) Assuming $|V_0^+| = 100$ V, how much average power is delivered to the load?

$$\begin{aligned} P_{L_{avg}} &= \frac{1}{2}\frac{|V_0^+|^2}{Z_0}(1-|\Gamma_L|^2) \\ &= \frac{1}{2}\frac{100^2}{50}(1-(0.2)^2) = 100(1-0.04) = 96 \text{ W} \end{aligned}$$

   (c) (6 pts) Find the load impedance.

$$\begin{aligned} \Gamma_L &= \frac{Z_L - Z_0}{Z_L + Z_0} \\ 0.2 &= \frac{Z_L - 50}{Z_L + 50} \rightarrow Z_L - 50 = 0.2Z_L + 10 \\ &\rightarrow 0.8Z_L = 60 \rightarrow Z_L = 60/0.8 = 75\ \Omega \end{aligned}$$

   (d) (8 pts) If the wave travels at the speed of light ($3\times10^8$ m/s) at a radian frequency of $\omega = 3\times10^9$ rad/s, write the *complete* expression for the voltage $V(z,t)$. Assume $V_0^+ = |V_0^+|$ from part (b), i.e., it's real. Don't forget to find $\beta$!

$$\beta = \frac{\omega}{u} = \frac{3\times10^9}{3\times10^8} = 10 \text{ rad/m}$$

$$V(z,t) = 100\cos(3\times10^9 t - 10z) + 20\cos(3\times10^9 t + 10z) \text{ V}$$

3. The voltage on a transmission line is given by:

$$V(z,t) = 5e^{-0.5z}\cos\left(4\times10^9 t - 26z + 0.4\right) + 2e^{0.5z}\cos\left(4\times10^9 t + 26z\right)\ \text{V}.$$

Use this expression to answer the following problems *by inspection when possible*. **Don't forget units!**

(a) (5 pts) Convert $V(z,t)$ to the phasor domain.

$$V_s(z) = 5e^{-(0.5+j26)z+j0.4} + 2e^{(0.5+j26)z}\ \text{V}$$

(b) (6 pts) What is the propagation constant?

$$\gamma = 0.5 + j26\ 1/\text{m}$$

(c) (4 pts) What is the maximum value of the incident voltage?

$$V_{max}^{inc} = 5\ \text{V}$$

(d) (4 pts) Can you find the SWR from this expression? Explain, but **don't try to find it!**.

Yes because we can find the reflection coefficient from the expression.

(e) (6 pts) What is the reflection coefficient (think very carefully!).

$$|\Gamma_L| = \frac{2}{5} = 0.4$$

$$0 = \phi + \theta_r = 0.4 + \theta_r \ \rightarrow\ \theta_r = -0.4$$

$$\Gamma_L = 0.4e^{-j0.4}$$

4. A lossless 100-$\Omega$ line is terminated by a load $Z_L = 80 + j50\ \Omega$. Choose the correct circle on the Smith chart on the following page, and use it to respond to the problems below. **Show your work on the Smith chart as directed.**

   (a) (5 pts) On the Smith chart, indicate and give the value of the normalized load impedance $z_L$.

   $$z_L = \frac{Z_L}{Z_0} = \frac{80 + j50}{100} = 0.8 + j0.5$$

   (b) (4 pts) On the Smith chart, indicate and give the value of the standing wave ratio. **No credit will be given for results found using an equation.**

   (c) (6 pts) On the Smith chart, indicate and give the value of the reflection coefficient. **No credit will be given for results found using an equation.**

   (d) (5 pts) What is the shortest distance in wavelengths from the load at which $Z_{in}$ is purely resistive? **Show the location on the Smith chart, but put your final answer here.**

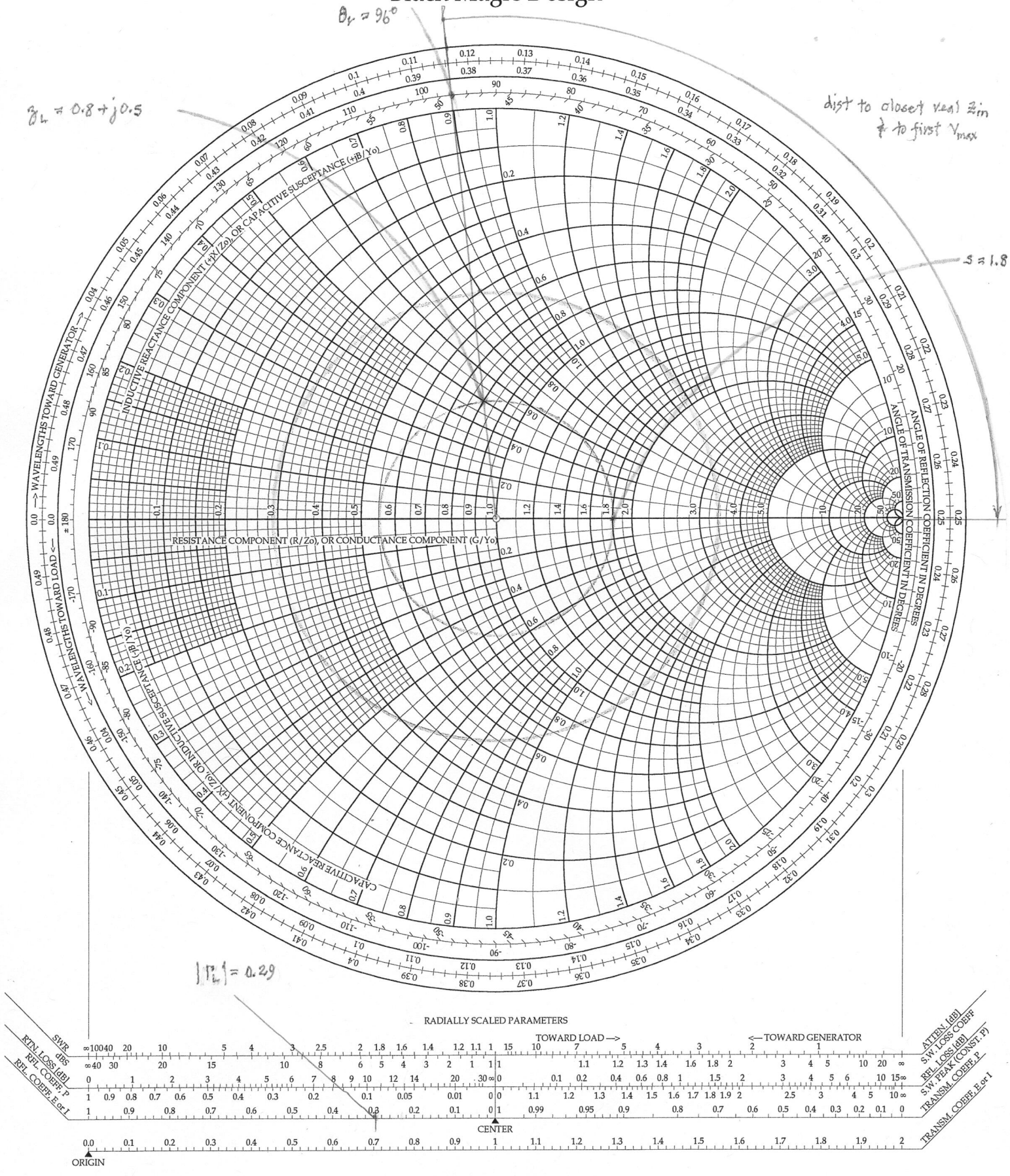
   $$0.25\lambda - 0.1165\lambda = 0.1335\lambda$$

   (e) (5 pts) What is the distance in wavelengths from the load to the first voltage maximum? **Show the location on the Smith chart, but put your final answer here.**

   $$0.25\lambda - 0.1165\lambda = 0.1335\lambda$$

# The Complete Smith Chart

## Black Magic Design




$\theta_r = 96°$

$z_L = 0.8 + j0.5$

dist to closest real $z_{in}$ = to first $V_{max}$

$S = 1.8$

$|\Gamma_L| = 0.29$