Name ______________________________

EE 331 – Exam #2
March 13, 2020
Closed Book/Closed Notes

Relax and think carefully. Show all your work to insure maximum partial credit. Remember that *neatness counts*! **SHOW ALL UNITS (e.g., V/m, m), and COMMUNICATE WHAT YOU KNOW!!**

1. (a) (4 pts) Given $\mathbf{A} = \frac{2}{r}\,\hat{\mathbf{a}}_r + \frac{3}{r}\,\hat{\mathbf{a}}_\phi$, find $\mathbf{A}\cdot\mathbf{dl}$.

$$\begin{aligned}\mathbf{A}\cdot\mathbf{dl} &= \left(\frac{2}{r}\,\hat{\mathbf{a}}_r + \frac{3}{r}\,\hat{\mathbf{a}}_\phi\right)\cdot\left(dr\,\hat{\mathbf{a}}_r + r\,d\theta\,\hat{\mathbf{a}}_\theta + r\sin\theta\,d\phi\,\hat{\mathbf{a}}_\phi\right)\\ &= \frac{2}{r}\,dr + 3\sin\theta\,d\phi\end{aligned}$$

(b) (8 pts) A vector field is given by $\mathbf{A} = \rho\sin\phi\,\hat{\mathbf{a}}_\phi$. Find the curl of $\mathbf{A}$.

$$\begin{aligned}\nabla\times\mathbf{A} &= \left[\frac{1}{\rho}\frac{\partial A_z}{\partial\phi} - \frac{\partial A_\phi}{\partial z}\right]\hat{\mathbf{a}}_\rho + \left[\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right]\hat{\mathbf{a}}_\phi + \frac{1}{\rho}\left[\frac{\partial(\rho A_\phi)}{\partial\rho} - \frac{\partial A_\rho}{\partial\phi}\right]\hat{\mathbf{a}}_z\\ &= \left[\frac{1}{\rho}\cancelto{0}{\frac{\partial A_z}{\partial\phi}} - \cancelto{0}{\frac{\partial A_\phi}{\partial z}}\right]\hat{\mathbf{a}}_\rho + \left[\cancelto{0}{\frac{\partial A_\rho}{\partial z}} - \cancelto{0}{\frac{\partial A_z}{\partial \rho}}\right]\hat{\mathbf{a}}_\phi + \frac{1}{\rho}\left[\frac{\partial(\rho A_\phi)}{\partial\rho} - \cancelto{0}{\frac{\partial A_\rho}{\partial\phi}}\right]\hat{\mathbf{a}}_z\\ &= \frac{1}{\rho}\left[\frac{\partial(\rho^2\sin\phi)}{\partial\rho}\right]\hat{\mathbf{a}}_z\\ &= 2\sin\phi\,\hat{\mathbf{a}}_z\end{aligned}$$

(c) (2 pts) In part (b), is $\mathbf{A}$ a conservative field? Explain (explanation should be short).

No, because $\nabla\times\mathbf{A}\neq 0$.

2. A vector field is given by $\mathbf{A} = 2x\,\widehat{\mathbf{a}}_x + 2y\,\widehat{\mathbf{a}}_y + (x^2y + z)\,\widehat{\mathbf{a}}_z$.

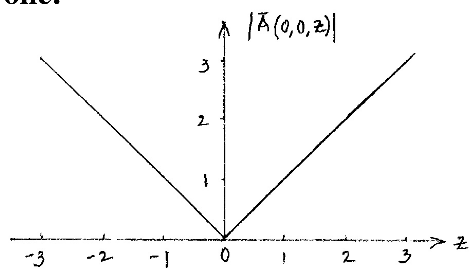
(a) (5 pts) Find the value of $\mathbf{A}$ at $\mathbf{r} = (1, 1, 1)$ m.

$$\begin{aligned}\mathbf{A}(1,1,1) &= 2(1)\,\widehat{\mathbf{a}}_x + 2(1)\,\widehat{\mathbf{a}}_y + ((1^2)(1)+1)\,\widehat{\mathbf{a}}_z \\ &= 2\,\widehat{\mathbf{a}}_x + 2\,\widehat{\mathbf{a}}_y + 2\,\widehat{\mathbf{a}}_z\end{aligned}$$

(b) (5 pts) Find the magnitude of $\mathbf{A}$, i.e., $|\mathbf{A}|$, in the $x = 0$ plane.

$$\begin{aligned}\mathbf{A}(0,y,z) &= 2y\,\widehat{\mathbf{a}}_y + z\,\widehat{\mathbf{a}}_z \\ |\mathbf{A}(0,y,z)| &= \sqrt{4y^2 + z^2}\end{aligned}$$

(c) (6 pts) Plot the magnitude of $\mathbf{A}$, i.e., $|\mathbf{A}|$, along the $z$ axis for $-3 \le z \le 3$ m. **Label your plot! Be careful! Think about this one!**




$$\begin{aligned}\mathbf{A}(0,0,z) &= z\widehat{\mathbf{a}}_z \\ |\mathbf{A}(0,0,z)| &= |z|\end{aligned}$$

(d) (9 pts) Use *the projection method* to transform $\mathbf{A}$ to *cylindrical* coordinates, **but just find** $A_\rho$.

$$\begin{aligned}A_\rho &= \mathbf{A}\cdot\widehat{\mathbf{a}}_\rho \\ &= 2x\,\widehat{\mathbf{a}}_x\cdot\widehat{\mathbf{a}}_\rho + 2y\,\widehat{\mathbf{a}}_y\cdot\widehat{\mathbf{a}}_\rho + (x^2y+z)\,\widehat{\mathbf{a}}_z\cdot\widehat{\mathbf{a}}_\rho \\ &= 2x\cos\phi + 2y\sin\phi + 0 \\ &= 2\rho\cos^2\phi + 2\rho\sin^2\phi \\ &= 2\rho\,(\cos^2\phi + \sin^2\phi) = 2\rho\end{aligned}$$

3. A vector field is given by $\mathbf{D} = \frac{2}{r}\,\widehat{\mathbf{a}}_r + \frac{3}{r}\,\widehat{\mathbf{a}}_\phi$ C/m$^2$. **Don't forget units.**

(a) (8 pts) Find the divergence of **D**.

$$\begin{aligned}
\nabla\cdot\mathbf{D} &= \frac{1}{r^2}\frac{\partial(r^2 D_r)}{\partial r} + 0 + \frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial\phi} \\
&= \frac{1}{r^2}\frac{\partial(2r)}{\partial r} + \frac{1}{r\sin\theta}\cancelto{0}{\frac{\partial}{\partial\phi}\left(\frac{3}{r}\right)} \\
&= \frac{2}{r^2}\ \text{C/m}^3
\end{aligned}$$

(b) (10 pts) Find the net outward flux $\oint_S \mathbf{D}\cdot\mathbf{dS}$ for a sphere of radius 2 m.

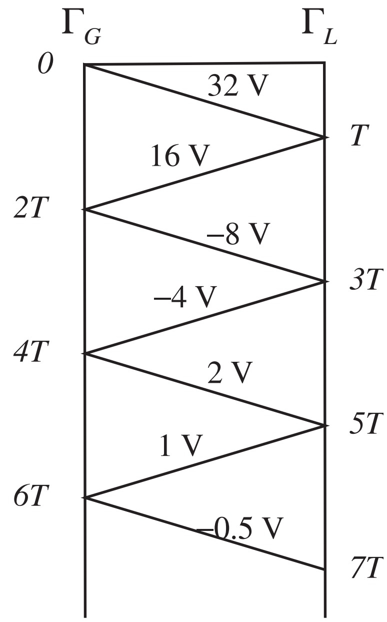
$$\begin{aligned}
\oint_S \mathbf{D}\cdot\mathbf{dS} &= \oint\left(\frac{2}{r}\,\widehat{\mathbf{a}}_r + \frac{3}{r}\,\widehat{\mathbf{a}}_\phi\right)\cdot r^2\sin\theta\,d\theta\,d\phi\,\widehat{\mathbf{a}}_r \\
&= 2r|_{r=2}\int_0^\pi \sin\theta\,d\theta\int_0^{2\pi} d\phi \\
&= 4\left(\cos\theta|_\pi^0\right)\left(\phi|_0^{2\pi}\right) \\
&= (4)(2)(2\pi) \\
&= 16\pi\ \text{C}
\end{aligned}$$

(c) (10 pts) Find the total charge in a sphere of radius 2 m if the volume charge density is $\rho_v = \frac{2}{r^2}$ C/m$^3$.

$$\begin{aligned}
\int_v \rho_v\,dv &= \int_v\left(\frac{2}{r^2}\right)r^2\sin\theta\,dr\,d\theta\,d\phi \\
&= 2\int_0^2 dr\int_0^\pi\sin\theta\,d\theta\int_0^{2\pi}d\phi \\
&= 2\left(r|_0^2\right)\left(\cos\theta|_\pi^0\right)\left(\phi|_0^{2\pi}\right) \\
&= (2)(2)(2)(2\pi) \\
&= 16\pi\ \text{C}
\end{aligned}$$

4. In the following problem $\Gamma_L = 1/2$, $\Gamma_G = -1/2$, and $V_1^+ = 32$ V.

   (a) (10 pts) Complete the bounce diagram by writing in the voltages $V_1^+$, $V_1^-$, $V_2^+$, and so on for all the diagonal lines in the bounce diagram.




   (b) (4 pts) What is $V_L$, the line voltage at the load, at $t = 0$ s?

$$V_L(0) = 0 \text{ V}$$

   (c) (4 pts) What is $V_G$, the line voltage at the generator, at $t = 2T$ s?

$$V_G(2T) = 32 + 16 - 8 = 40 \text{ V}$$

5. A 50-Ω transmission line is connected to an antenna with a normalized load impedance as shown on the Smith chart on the next page. **Show your work on the Smith chart as directed.**

   (a) (5 pts) From the Smith chart, find the normalized load admittance $y_L$. Indicate and give the value of $y_L$ on the Smith chart.

   (b) (5 pts) Indicate and give the **two** possible values for $y_d$ (the admittance of the load and the line) on the Smith chart.

   (c) (5 pts) Indicate and give the **two** possible values for $y_s$ (the admittance of the stub tuner) on the Smith chart.

# The Complete Smith Chart

Black Magic Design