Homework-11

4.60 (Book)

1) An electric dipole with $\bar{p} = p a_z$ C.m is placed at (0,0,0).
Potential at (0, 0, 1 nm) = 9V.

Potential at point $P(r, \theta, \phi)$ is given by

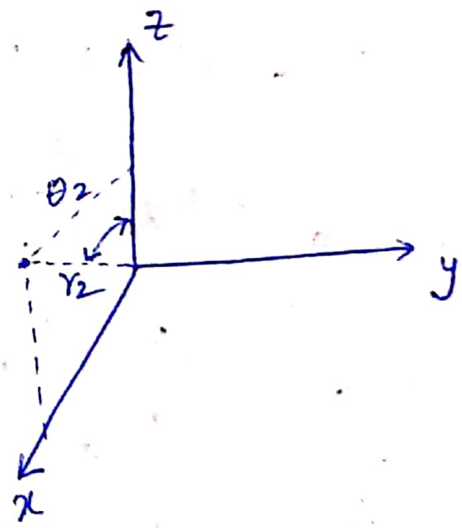
$$V = \frac{\bar{P} \cdot \hat{a_r}}{4\pi\epsilon_0 r^2} = \frac{p\,\hat{a_z} \cdot \hat{a_r}}{4\pi\epsilon_0 r^2} = \frac{p\cos\theta}{4\pi\epsilon_0 r^2} \quad V \rightarrow (1).$$

Let, $\frac{p}{4\pi\epsilon_0 r^2} = k$.

At (0, 0, 1nm) $V_1 = 9V$, $\theta_1 = 0°$, $r_1 = 1$ nm. (1) can be written as →

$$V_1 = \frac{k\cos\theta_1}{r_1^2} = 9 \Rightarrow k = \frac{9 r_1^2}{\cos\theta_1} \rightarrow (2)$$

At (1 nm, 0, 1nm), $\theta_2 = 45°$, $r_2 = \sqrt{2}$ nm.



$$V_2 = \frac{k\cos\theta_2}{r_2^2}$$

$$= \frac{9 r_1^2}{\cos\theta_1} \cdot \frac{\cos\theta_2}{r_2^2}$$

$$= \frac{9 \cdot 1^2}{\cos 0°} \cdot \frac{\cos(\pi/4)}{(\sqrt{2})^2} = \frac{9}{2\sqrt{2}} = 3.1820 \text{ V}.$$

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2.) Prob 4.66 (In Book)

Energy in hemisphere $r \le 2m$, $0 < \theta < \pi$, $0 < \phi < \pi$.

$$\bar{E} = 2r\sin\theta\cos\phi\, \hat{a_r} + r\cos\theta\cos\phi\, \hat{a_\theta} - r\sin\phi\, \hat{a_\phi} \text{ V/m.}$$

$$W_E = \frac{1}{2}\epsilon_0 \int |E|^2 dv.$$

$$|E|^2 = 4r^2\sin^2\theta\cos^2\phi + r^2\cos^2\theta\cos^2\phi + r^2\sin^2\phi$$
$$= 3r^2\sin^2\theta\cos^2\phi + r^2\sin^2\theta\cos^2\phi + r^2\cos^2\theta\cos^2\phi + r^2\sin^2\phi$$
$$= 3r^2\sin^2\theta\cos^2\phi + r^2\cos^2\phi + r^2\sin^2\phi$$
$$= r^2(3\sin^2\theta\cos^2\phi + 1)$$

$dv = r^2\sin\theta\, dr\, d\theta\, d\phi$

$$W_E = \frac{1}{2}\epsilon_0 \int r^2(1 + 3\sin^2\theta\cos^2\phi)\, r^2\sin\theta\, dr\, d\theta\, d\phi$$
$$= \frac{1}{2}\epsilon_0 \int_0^2 r^4\, dr \int_0^\pi \sin\theta\, d\theta \int_0^\pi (1 + 3\sin^2\theta\cos^2\phi)\, d\phi$$
$$= \frac{1}{2}\epsilon_0 \left[\frac{r^5}{5}\right]_0^2 \int_0^\pi \sin\theta\, d\theta \left[[\phi]_0^\pi + \left[3\sin^2\theta\left(\frac{\phi}{2} + \frac{\sin 2\phi}{4}\right)\right]_0^\pi \right.$$
$$= \frac{16\epsilon_0}{5} \int_0^\pi \sin\theta \left\{ \pi + 3\sin^2\theta\,(\pi/2 + 0)\right\} d\theta$$
$$= \frac{16\epsilon_0}{5} \int_0^\pi \left(\pi\sin\theta + 3\sin^3\theta.\frac{\pi}{2}\right) d\theta$$
$$= \frac{16\epsilon_0}{5} \left\{ \left[\pi\cos\theta\right]_\pi^0 + \frac{3\pi}{2}\left[-\cos\theta + \frac{\cos^3\theta}{3}\right]_0^\pi \right\}.$$

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$$= \frac{16\epsilon_0}{5}\left(2\pi + \frac{3\pi}{2}\cdot\frac{4}{3}\right)$$

$$= \frac{16\epsilon_0}{5}\cdot 4\pi = \frac{64\pi\epsilon_0}{5} = 356.0405 \text{ p Joule.}$$

3) Prob 5.4

In a cylindrical conductor of radius 4 mm, the current density is $J = 5e^{-10\rho}\,\hat{a_z}$ A/m^2.

$$I = \int_S \bar{J}.d\bar{s}$$

Since $\bar{J}$ is directed towards $\hat{a_z}$, the differential surface $ds_z = \rho\, d\rho\, d\phi\, \hat{a_z}$ will give a ~~a~~ non-zero dot product.

$$= \int 5e^{-10\rho}\hat{a_z}.\,\rho\, d\rho\, d\phi\, \hat{a_z}$$

$$= 5\int_0^{2\pi} d\phi \int_0^{0.004} \rho\, e^{-10\rho}\, d\rho.$$

$$= 5\left\{[\phi]_0^{2\pi}\left[\left(-\frac{\rho}{10} - \frac{1}{100}\right)e^{-10\rho}\right]_0^{0.004}\right.$$

$$= 5\times 2\pi \times 7.7898\times 10^{-6} = 244.7248\ \mu A.$$

$$\int \rho e^{-10\rho} d\rho = \rho\int e^{-10\rho} - \int\left\{\frac{\partial}{\partial\rho}(\rho)\int e^{-10\rho}\right\}d\rho$$

$$= -\frac{\rho}{10}e^{-10\rho} - \int \frac{e^{-10\rho}}{(-10)}d\rho$$

$$= -\frac{\rho}{10}e^{-10\rho} - \frac{1}{100}e^{-10\rho}$$

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4)

Magic wand → cylindrical.

σ (conductivity) = 6.2×10^7 S/m.

radius = 0.5 cm.

Length = 0.5 m.

When an electric field of 12 mV/m is applied to it, 10^{19} free electrons per cubic meter are available.

(a) Volume charge density

$$\rho_v = ne = 10^{19} \times (-1.6 \times 10^{-19}) = -1.6 \ C/m^3.$$

(b) Current density

$$\bar{J} = \sigma \bar{E} = 6.2 \times 10^7 \times 12 \times 10^{-3} = 74.4 \times 10^4 \ A/m^2$$

$$|\bar{J}| = 744 \ kA/m^2$$

(c) Current flowing through the wand

$$I = JS = J\pi r^2 = 744 \times 10^3 \times \pi \times (0.005)^2 = 58.433 A$$

(d) Drift velocity of electrons in the wand

$$u = \frac{J}{|\rho_v|} = \frac{744 \times 10^3}{|-1.6|} = 465 \ km/s.$$

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(e) Resistance of the wand

$$R = \frac{l}{\sigma S} = \frac{0.5}{6.2\times10^{7}\times\pi\times(0.005)^2} = 102.68\ \mu\Omega.$$

(f) Amount of power flowing through the wand (with 12mV/m electric field)

$$P = \int_V \bar{J}.\bar{E}\,dv = \int_V \sigma|\bar{E}|^2 dv.$$

$\bar{J} = \sigma\bar{E}$

$\bar{J}.\bar{E} = \sigma|\bar{E}|^2$.

$$= (12\times10^{-3})^2 \times 6.2\times10^{7}\times\pi\times(0.005)^2 \times 0.5$$

$$= 0.3506 = 350.6\ mW.$$

$$P = I^2R = (58.433)^2 \times 102.68\times10^{-6} = 0.35059 \approx 350.6\ mW.$$

5) Pumpkin pie has dielectric constant ($\epsilon_r = 65$) at microwave frequencies.

(a) Electric susceptibility

$$\chi_e = \epsilon_r - 1 = 65 - 1 = 64.$$

(b) Permittivity

$$\epsilon = \epsilon_r\epsilon_0 = 65\times8.54\times10^{-12} = 575.5\ pF/m.$$

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Electric flux density

(e) $\overline{D} = \epsilon \overline{E}$

$= (575 \times 10^{-12}) \times 10^3 = 575.51\, nC/m^2$

$|\overline{D}| = 575.51\, nC/m^2$

6:>

Prob 5.34b

$J = 0.5 \sin \pi x \ \hat{a_x} \cdot A/m^2$.

Time rate of increase of charge density $\left(\frac{\partial \rho_v}{\partial t}\right)$.

$$\nabla \cdot \overline{J} = -\frac{\partial \rho_v}{\partial t}$$

$$\nabla \cdot \overline{J} = \frac{\partial}{\partial x}(0.5 \sin \pi x) = 0.5\pi \cos \pi x$$

Time rate of increase of charge density $\left(\frac{\partial \rho_v}{\partial t}\right)$ at $(2,4,-3)$

$$= -0.5\pi \cos \pi x \Big|_{x=2}$$

$$= -0.5\pi \cos 2\pi = -0.5\pi \ C/m^3 s.$$

Prob 5.31 (a)

$$\overline{J} = \frac{100}{\rho^2} \hat{a_\rho} \cdot A/m^2.$$

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$$\nabla \cdot \bar{J} = -\frac{\partial \rho_v}{\partial t}$$

$$\frac{\partial \rho_v}{\partial t} = -\nabla \cdot \bar{J}$$

$$= -\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \cdot \frac{100}{\rho^2}\right)$$

$$= -\frac{1}{\rho}\frac{\partial}{\partial \rho}\cdot\left(\frac{100}{\rho}\right) = \frac{100}{\rho^3} \cdot \; C/m^3 s.$$

Prob 5.35.) (a) Hard rubber ($\sigma = 10^{-15}$ s/m, $\epsilon = 3.1\epsilon_0$).

Relaxation Time $(T_r) = \frac{\epsilon}{\sigma} = \frac{3.1 \times 8.854 \times 10^{-12}}{10^{-15}}$ seconds

$= 27.44 \times 10^{3}$ s.

(b) Mica

$T_r = \frac{\epsilon}{\sigma} = \frac{6 \times 8.854 \times 10^{-12}}{10^{-15}} = 53.124 \times 10^{3}$ s.

(c) Distilled water

$T_r = \frac{\epsilon}{\sigma} = \frac{80 \times 8.854 \times 10^{-12}}{10^{-4}} = 708.32 \times 10^{-8}$ s

$= 7.08\ \mu s.$

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7.) Lightning strikes a dielectric sphere of radius 20mm for which $\epsilon_r = 2.5$, $\sigma = 5\times10^{-6}$ S/m and deposits a charge of 1C.

Initial volume charge density $(\rho_{v0}) = \frac{Q}{v}$

$$= \frac{1}{4/3\,\pi r^3}$$

$$= \frac{1}{4/3\,\pi \times (20\times10^{-3})^3}$$

$$= 29.84 \text{ kC/m}^3.$$

Relaxation Time $(T_r) = \frac{\epsilon}{\sigma}$

$$= \frac{\epsilon_r \epsilon_0}{\sigma} = \frac{2.5\times 8.854\times10^{-12}}{5\times10^{-6}}$$

$$= 4.427\times10^{-6} \text{ seconds.}$$

Volume charge density after $2\mu s$ $(\rho_v) = \rho_{v0}\, e^{-t/T_r}\Big|_{t=2\mu s}$

$$= 29.84\times10^{3}\, e^{-2/4.427}$$

$$= 18.993 \text{ kC/m}^3.$$

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