1.) Ch5 Prob 38(b)

a)

Region 1 $(z<0)$, $\epsilon_{r_1}=4$, $\bar{E}_1 = 60\hat{a_x} - 100\hat{a_y} + 40\hat{a_z}$ V/m.

Region 2 $(z>0)$ $\epsilon_{r_2} = 7.5$

Boundary conditions $\Rightarrow$ $E_{1t} = E_{2t}$ $\qquad \bar{D} = \epsilon\bar{E}$.

$D_{1n} = D_{2n}$

$\bar{E}_{1t} = 60\hat{a_x} - 100\hat{a_y} = \bar{E}_{2t}$

$\therefore \bar{D}_{2t} = \epsilon_2 \bar{E}_{2t} = \epsilon_{r_2}\cdot\epsilon_0\cdot \bar{E}_{2t}$

$= 7.5\epsilon_0 (60\hat{a_x} - 100\hat{a_y})$.

$\bar{E}_{1n} = 40\hat{a_z} \Rightarrow \bar{D}_{1n} = \epsilon_1 \bar{E}_{1n}$

$= 4\epsilon_0 \cdot 40\hat{a_z}$.

$\bar{D}_2 = \bar{D}_{2t} + \bar{D}_{1n} = 7.5\epsilon_0 (60\hat{a_x} - 100\hat{a_y}) + 4\epsilon_0 \cdot 40\hat{a_z}$ C/m^2

$= (3.984\hat{a_x} - 6.64\hat{a_y} + 1.416\,\hat{a_z})$ nC/m^2.

(b) Prob 43(a)

Region1 ($\rho \leq 4\,cm$) $\epsilon_{r_1} = 3.5$

Region 2 ($\rho \geq 4\,cm$) $\epsilon_{r_2} = 1.5$ $\bar{D}_2 = 12\hat{a}_\rho - 6\hat{a}_\phi + 9\hat{a}_z$ nC/m^2

Boundary conditions $E_{1t} = E_{2t}$ $\bar{D} = \epsilon\bar{E}$.

$D_{1n} = D_{2n}$

$$\bar{D}_{2t} = -6\hat{a}_\phi + 9\hat{a}_z$$

$$\bar{E}_{2t} = \frac{1}{\epsilon_2}\bar{D}_{2t} = \frac{1}{1.5\epsilon_0}(-6\hat{a}_\phi + 9\hat{a}_z)\times 10^{-9}$$

$$\bar{E}_{1t} = \bar{E}_{2t} = \frac{1}{1.5\epsilon_0}(-6\hat{a}_\phi + 9\hat{a}_z)\times 10^{-9}.$$

$$\bar{D}_{2n} = 12\hat{a}_\rho \text{ nC/m}^2 = \bar{D}_{1n}$$

$$\bar{E}_{1n} = \frac{1}{\epsilon_1}\bar{D}_{1n} = \frac{1}{3.5\epsilon_0}\cdot 12\hat{a}_\rho \times 10^{-9}$$

$$\bar{E}_1 = \bar{E}_{1t} + \bar{E}_{1n} = \left[\frac{1}{1.5\epsilon_0}(-6\hat{a}_\phi + 9\hat{a}_z) + \frac{1}{3.5\epsilon_0}12\hat{a}_\rho\right]\times 10^{-9}$$

$$= (387.23\,\hat{a}_\rho - 451.77\,\hat{a}_\phi + 677.65\,\hat{a}_z)\text{ V/m.}$$

2a) Prob 6.36

$\epsilon_r = 4.$ $d = \cancel{\ldots}$ 1 μm. $C = 2nF$

$$C = \frac{\epsilon S}{d} \Rightarrow S = \frac{Cd}{\epsilon} = \frac{2\times10^{-9}\times10^{-6}}{4\times8.854\times10^{-12}}$$

$$= 56.47\times10^{-6} m^2 = 56.47\ cm^2.$$

2b) $\epsilon_r = 2.$ Inner radius(a) = 5 mm. Outer radius (b) = 10 mm

$$C = \frac{2\pi\epsilon L}{\ln b/a} \Rightarrow \frac{C}{L} = \frac{2\pi\epsilon}{\ln b/a}$$

$$= \frac{2\pi\times2\times8.854\times10^{-12}}{\ln(10/5)}$$

$$= 160.51\ pF/m.$$

3.) Example 2.4 with b = 10 mm

Cylindrical capacitor has inner and outer radii of a = 5 mm, ~~…~~ b = 10 mm

$V(a) = 100V.$ $V(b) = 0V$ $\epsilon_r = 2.$

No charge density $\Rightarrow \rho_v = 0.$

$$\therefore \nabla^2 V = 0.$$

$$\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right) = 0.$$

$$\rho \frac{\partial v}{\partial \rho} = A$$

$$\Rightarrow \frac{\partial v}{\partial \rho} = \frac{A}{\rho}$$

$$\Rightarrow V = A\ln(\rho) + B.$$

$$V\Big|_{\rho = 0.010} = A\ln(0.010) + B = 0 \Rightarrow B = -A\ln(0.01)$$

$$V\big|_{\rho=0.005} = A\ln(0.005) + B$$

$$= A\ln(0.005) - A\ln(0.01)$$

$$= A\ln(1/2).$$

$$V\big|_{\rho=0.005} = 100.$$

$$\therefore A\ln(0.5) = 100$$

$$\Rightarrow A = \frac{100}{\ln(0.5)} = \frac{-100}{\ln(2)}$$

$$\therefore V = \frac{-100}{\ln(2)}\ln(\rho) + \frac{100}{\ln(2)}\ln(0.01)$$

$$= \frac{-100}{\ln(2)}\ln\left(\frac{\rho}{0.01}\right) V = \frac{-100}{\ln(2)}\ln(100\rho)\ V.$$

$$\bar{E} = -\nabla \bar{V} = -\frac{\partial V}{\partial \rho}\hat{a}_\rho = \frac{100}{\ln(2)\rho}\hat{a}_\rho \ V/m.$$

$$= \frac{144.269}{\rho}\hat{a}_\rho \ V/m.$$

$$\bar{D} = \epsilon\bar{E} = \epsilon_r\epsilon_0\bar{E} = 2\times 8.854\times 10^{-12}\times\frac{144.269}{\rho}\hat{a}_\rho \ V/m$$

$$= \frac{2.554}{\rho}\ nC/m^2 = \frac{200\epsilon_0}{\ln(2)\rho}\hat{a}_\rho \ C/m^2.$$

Boundary Conditions

$$\rho_s(\rho = 0.005) = D(\rho = 0.005) = \frac{200\epsilon_0}{\ln(2)\times 0.005} = 510.944\ nC/m^2.$$

$$\rho_s(\rho = 0.01) = D(\rho = 0.01) = \frac{-200\epsilon_0}{\ln(2)\times 0.01} = -255.47\ nC/m^2.$$

find Q

$$Q = \rho_s \cdot S = \rho_s \cdot 2\pi\rho L = 510.944\times 10^{-9}\times 2\pi\times 0.005\times L$$

$$= 16.0517\ L\ nC$$

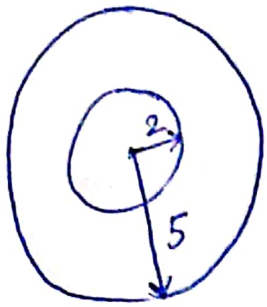
$$Q/V = C = \frac{16.0517L\times 10^{-9}}{100} = 160.5780\ L\ PF.$$

$$C/L = 160.5781\ PF/m.$$

$\downarrow$

This is same as Prob 2(b)

4) Prob 6.7



$$\nabla^2 V = -\rho_v/\epsilon$$

$$\Rightarrow \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial v}{\partial\rho}\right) = -\frac{10\times10^{-12}}{\rho\cdot 3.6\epsilon_0}$$

$$\Rightarrow \frac{\partial}{\partial\rho}\left(\rho\frac{\partial v}{\partial\rho}\right) = \frac{-10^{-11}}{3.6\epsilon_0}.$$

$$\Rightarrow \rho\frac{\partial v}{\partial\rho} = \frac{-10^{-11}\rho}{3.6\epsilon_0} + A.$$

$$\Rightarrow \frac{\partial v}{\partial\rho} = \frac{-10^{-11}}{3.6\epsilon_0} + \frac{A}{\rho}$$

$$\Rightarrow V = \frac{-10^{-11}}{3.6\epsilon_0}\cdot\rho + A\ln|\rho| + B.$$

Boundary condition Values

$$V(\rho=2) = 0\,V$$

$$V(\rho=5) = 60.$$

$$V(\rho=2) = \frac{-10^{-11}}{3.6\epsilon_0}\cdot 2 + A\ln|2| + B.$$

$$\Rightarrow B = \frac{2\times10^{-11}}{3.6\epsilon_0} - A\ln|2| \longrightarrow (1).$$

Prob-7

$$V(\rho=5) = \frac{-10^{-11}}{3.6\epsilon_0}\cdot 5 + A\ln|5| + B.$$

$$\Rightarrow 60 = \frac{-5\times10^{-11}}{3.6\epsilon_0} + A\ln|5| + \frac{2\times10^{-11}}{3.6\epsilon_0} - A\ln|2|$$

$$\Rightarrow 60 = \frac{-3\times10^{-11}}{3.6\epsilon_0} + A\ln|2.5|.$$

$$\Rightarrow \quad A = 66.5085.$$

$$B = \frac{2\times10^{-11}}{3.6\epsilon_0} - 66.5085\ln|2|$$

$$\therefore B = -45.472$$

$$\cancel{V = \frac{-10^{-11}}{3.6\epsilon_0} \times 66.5085}$$

$$V = \frac{-10^{-11}}{3.6\epsilon_0}\rho + 66.5085\ln|\rho| - 45.472$$

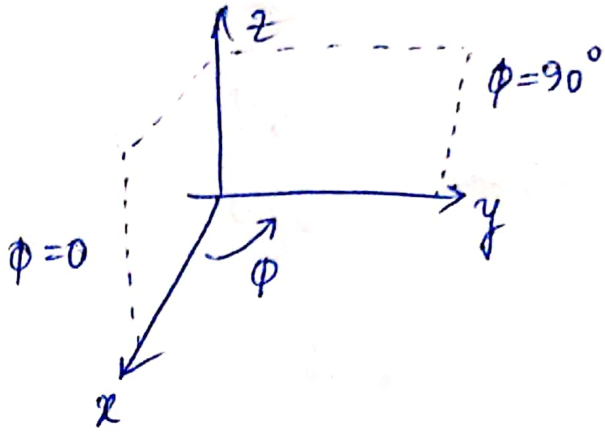
$$V = -0.3137\rho + 66.5085\ln|\rho| - 45.472$$

$$\bar{E} = -\nabla\bar{V} = -\frac{\partial V}{\partial\rho}\hat{a}_\rho = \left(0.3137 - \frac{66.6085}{\rho}\right)\hat{a}_\rho \text{ V/m.}$$

Scanned with CamScanner

5.) Prob 6.16

In cylindrical coordinates, $V = 50V$ on plane $\phi = \pi/2$,
$V = 0$ on plane $\phi = 0$.



$\nabla^2 V = 0.$

Since, V is independent of z & ρ,

$$\frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} = 0.$$

$$\Rightarrow \frac{\partial^2 V}{\partial \phi^2} = 0.$$

$$\Rightarrow \frac{\partial V}{\partial \phi} = A$$

$$V = A\phi + B.$$

Boundary values $V(\phi = 0°) = 0.$
$V(\phi = \pi/2) = 50.$

$$V(\phi = 0) = A.0 + B \Rightarrow B = 0.$$

$$V(\phi = \pi/2) = A.\pi/2$$

$$\Rightarrow \frac{\pi}{2} A = 50 \Rightarrow A = \frac{100}{\pi}$$

$V = \frac{100}{\pi}\phi$ Volts.

$\bar{E} = -\nabla \bar{V} = -\frac{1}{\rho}\frac{\partial V}{\partial \phi}\hat{a\phi} = \frac{-100}{\pi\rho}\hat{a\phi}$ V/m.

6.) Redo Example #26, $a = 3m$, $b = 6m$, $V_0 = 5V$.

The general solution is given by →

$$V(x,y) = \sum_{n=1}^{\infty} C_n \sinh\left(\frac{n\pi}{a}x\right)\sin\left(\frac{n\pi}{a}y\right) \text{ V.}$$

Apply Boundary Values

$$V_0 = V(b,y) = \sum_{n=1}^{\infty} C_n \sinh\left(\frac{n\pi b}{a}\right)\sin\left(\frac{n\pi}{a}y\right)$$

$$\Rightarrow 5 = \sum_{n=1}^{\infty} C_n \sinh(2n\pi)\sin\left(\frac{n\pi}{3}y\right)$$

Multiply both sides by ~~$\sin\left(\frac{n\pi}{3}y\right)$~~ $\sin\left(\frac{m\pi}{3}y\right)$ and integrate

$$\int_0^3 5\sin\left(\frac{m\pi}{3}y\right)dy = \int_0^3 \sum_{n=1}^{\infty} C_n \sinh(2n\pi)\sin\left(\frac{n\pi}{3}y\right)\sin\left(\frac{m\pi}{3}y\right)dy.$$

$$= \sum_{n=1}^{\infty} C_n \sinh(2n\pi)\int_0^3 \sin\left(\frac{n\pi}{3}y\right)\sin\left(\frac{m\pi}{3}y\right)dy$$

$$= \frac{3}{2} C_n \sinh(2n\pi).$$

Look at left hand side,

$$5\int_0^3 \sin\left(\frac{n\pi}{3}y\right)dy = 5\left(\frac{3}{n\pi}\right)\cos\left(\frac{n\pi}{3}y\right)\Big|_3^0$$

$$= \frac{15}{n\pi}\left(1-\cos(n\pi)\right).$$

if n is even,

$$\left(1-\cos(n\pi)\right)=0$$

if n is odd, $\left(1-\cos(n\pi)\right)=2.$

$$\therefore LHS = \cancel{30}\ \frac{30}{n\pi} = RHS = C_n \sinh(2n\pi)\cdot\frac{3}{2}$$

$$\Rightarrow C_n \sinh(2n\pi) = \frac{20}{n\pi}$$

$$\therefore C_n = \frac{20}{\pi}\cdot\frac{1}{n\sinh(2n\pi)}$$

finally,

$$V(x,y) = \frac{20}{\pi}\sum_{n=odd}\frac{1}{n\sinh(2n\pi)}\sinh\left(\frac{n\pi}{3}x\right)\sin\left(\frac{n\pi}{3}y\right) \text{ Volts.}$$