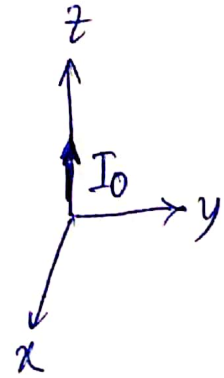
# Homework 13

1.)




Differential length $(dl) \Rightarrow \bar{dl}' = dz'\hat{a_z}$ m.

$\bar{r} = \rho\hat{a_\rho}$ m.

$\bar{r}' = z'\hat{a_z}$ m.

$\bar{r} - \bar{r}' = \rho\hat{a_\rho} - z'\hat{a_z}$ m.

$|\bar{r} - \bar{r}'|^3 = \left[z'^2 + \rho^2\right]^{3/2}$ $m^3$

$\bar{dl} \times (\bar{r} - \bar{r}') = dz'\hat{a_z} \times (\rho\hat{a_\rho} - z'\hat{a_z})$

$= \rho dz'\hat{a_\phi}$ $m^2$.

## Biot-Savart's Law

$$\bar{H}(\bar{r}) = \frac{1}{4\pi}\int \frac{I\bar{dl}' \times (\bar{r} - \bar{r}')}{|\bar{r} - \bar{r}'|^3} = \frac{I_0}{4\pi}\int_{-\infty}^{\infty} \frac{\rho dz'\hat{a_\phi}}{\left[z'^2 + \rho^2\right]^{3/2}}$$

(Since it is an even function)

$$= \frac{I_0 . 2}{4\pi}\int_0^{\infty} \frac{\rho dz'\hat{a_\phi}}{\left[z'^2 + \rho^2\right]^{3/2}}$$

$$= \frac{I_0}{2\pi}\hat{a_\phi}\int_0^{\infty} \frac{dz'}{(z'^2 + \rho^2)^{3/2}}$$

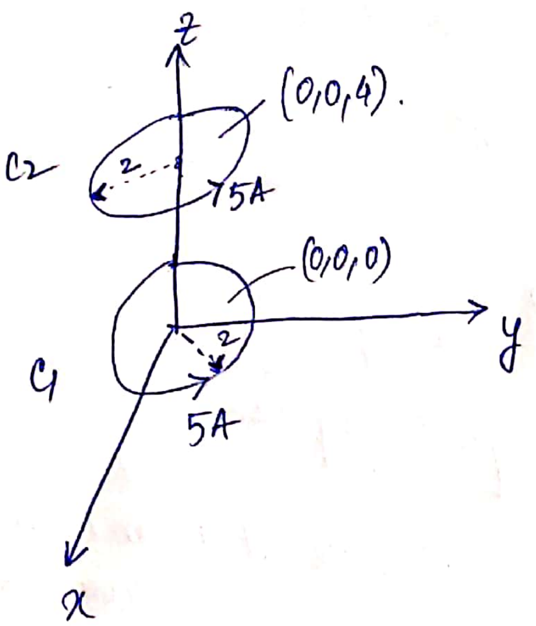
$$= \frac{I_0}{2\pi} . \hat{a_\phi} . \left| \frac{z'}{\rho^2\sqrt{z'^2 + \rho^2}} \right|_0^{\infty}.$$

$$\lim_{z'\to\infty} \frac{z'}{\rho^2\sqrt{z'^2+\rho^2}} = \lim_{z'\to\infty} \frac{1}{\rho^2\sqrt{1+\rho^2/z'^2}}$$ Using L'Hospital rule.

$$= \frac{1}{\rho^2}.$$

$$\therefore\ \bar{H}(\bar{r}) = \frac{I_0 \cdot \rho}{2\pi} \cdot \frac{1}{\rho^2}\, \hat{a_\varphi} = \frac{I_0}{2\pi\rho}\, \hat{a_\varphi} \text{ A/m.}$$

2.)




$I_{C1} = I_{C2} = 5A.$

a) $\bar{H}$ at $(0,0,0)$. $\bar{r}$ (Position vector at origin) $= (0,0,0)$.

$\bar{r}_1'$ (Position vector for coil $C_1$) $= 2\hat{a_\rho}$ m.

$\bar{r}_2'$ (Position vector for coil $C_2$) $= 2\hat{a_\rho} + 4\hat{a_z}$ m.

$\bar{r} - \bar{r}_1' = -2\hat{a_\rho}$ m.

$\bar{r} - \bar{r}_2' = -2\hat{a_\rho} - 4\hat{a_z}$ m.

Differential length $(dl')$

$\bar{dl}' = 2d\varphi' \hat{a_\varphi}$ m.

$\bar{dl} \times (\bar{r} - \bar{r}_1) = 2d\varphi' \hat{a_\varphi} \times (-2\hat{a_\rho}) = 4d\varphi' \hat{a_z}$ m$^2$

$\bar{dl} \times (\bar{r} - \bar{r}_2) = 2d\varphi' \hat{a_\varphi} \times (-2\hat{a_\rho} - 4\hat{a_z}) = 4d\varphi' \hat{a_z} - 8d\varphi' \hat{a_\rho}.$

$$|\bar{r}-\bar{r}_1'|^3 = 4^{3/2} = 8\,m.$$

$$|\bar{r}-\bar{r}_2'|^3 = (2^2+4^2)^{3/2} = (20)^{3/2} = (4\times5)^{3/2} = 8\times5\sqrt{5} = 40\sqrt{5}\ m.$$

$\bar{H}(0,0,0) = \bar{H}(0,0,0)$ due to $C_1$ + $\bar{H}(0,0,0)$ due to $C_2$.

$$= \frac{1}{4\pi}\int \frac{I_{C_1}\, d\bar{l}_1 \times (\bar{r}-\bar{r}_1')}{\cancel{(r_1'^2+r_1'^2)^{3/2}}\ |\bar{r}-\bar{r}_1'|^3} + \frac{1}{4\pi}\int \frac{I_{C_2}\, d\bar{l}_1 \times (\bar{r}-\bar{r}_2')}{|\bar{r}-\bar{r}_2'|^3}$$

$$= \frac{1}{4\pi}\left[\int_0^{2\pi} \frac{5.4\,d\phi'\hat{a}_z}{8} + \int_0^{2\pi} \frac{5(4\,d\phi'\hat{a}_z - \overset{0}{\cancel{8\,d\phi'\hat{a}_\rho}})}{40\sqrt{5}}\right]$$ due to symmetry

$$= \frac{1}{4\pi}\left[\frac{5}{2}\hat{a}_z\int_0^{2\pi} d\phi' + \frac{1}{2\sqrt{5}}\hat{a}_z\int_0^{2\pi} d\phi'\right]$$

$$= \frac{1}{4\pi}\left[\frac{5}{2}\cdot 2\pi\,\hat{a}_z + \frac{1}{2\sqrt{5}}\cdot 2\pi\cdot\hat{a}_z\right].$$

$$= \frac{1}{2}\left(\frac{5}{2}+\frac{1}{2\sqrt{5}}\right)\hat{a}_z = 1.3618\,\hat{a}_z \ \ A/m$$

b) $\bar{H}$ at $(0,0,2)$

$\bar{r}$ (Position vector at $(0,0,2)$) $= 2\hat{a}_z$ m.

$\bar{r}_1' = 2\hat{a}_\rho$ m.

$\bar{r}_2' = 2\hat{a}_\rho + 4\hat{a}_z$ m.

$\bar{r} - \bar{r}_1' = -2\hat{a}_\rho + 2\hat{a}_z$ m.

$\bar{r} - \bar{r}_2' = -2\hat{a}_\rho - 2\hat{a}_z$ m.

$d\bar{l} \times (\bar{r} - \bar{r}_1') = 2d\phi'\hat{a}_\phi \times (-2\hat{a}_\rho + 2\hat{a}_z)$

$= 4d\phi'\hat{a}_z + 4d\phi'\hat{a}_\rho.$

$d\bar{l} \times (\bar{r} - \bar{r}_2') = 2d\phi'\hat{a}_\phi \times (-2\hat{a}_\rho - 2\hat{a}_z)$

$= 4d\phi'\hat{a}_z - 4d\phi'\hat{a}_\rho.$

$|\bar{r} - \bar{r}_1'|^3 = (4+4)^{3/2} = 8^{3/2}$

$|\bar{r} - \bar{r}_2'|^3 = 8^{3/2}.$

$\bar{H}(0,0,2) = \bar{H}(0,0,2)$ due to coil $C_1$ $+ \bar{H}(0,0,2)$ due to coil $C_2$.

$$= \frac{1}{4\pi}\int \frac{I_{C_1} \cdot d\bar{l} \times (\bar{r} - \bar{r}_1')}{|\bar{r} - \bar{r}_1'|^3} + \frac{1}{4\pi}\int \frac{I_{C_2}\, d\bar{l} \times (\bar{r} - \bar{r}_2')}{|\bar{r} - \bar{r}_2'|^3}$$

due to symmetry.

$$= \frac{1}{4\pi}\int_{\phi=0}^{\phi=2\pi} \frac{5(4d\phi'\hat{a}_z + \cancel{4d\phi'\hat{a}_\rho})^{0}}{8^{3/2}} + \frac{1}{4\pi}\int_{\phi=0}^{\phi=2\pi} \frac{5(4d\phi'\hat{a}_z - \cancel{4d\phi'\hat{a}_\rho})}{8^{3/2}}$$

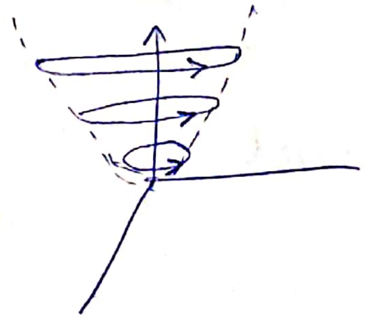
$$= \left(\frac{1}{4\pi} \cdot \frac{20}{8^{3/2}} \cdot 2\pi + \frac{1}{4\pi} \cdot \frac{20}{8^{3/2}} \cdot 2\pi\right)\hat{a}_z = 0.8839\,\hat{a}_z \text{ A/m}.$$

3) Prob 7.27

$$\bar{H} = 10^3 \rho^2 \hat{a}_\phi \text{ A/m.}$$

$$\Rightarrow \bar{H} = H_\phi \hat{a}_\phi \text{ A/m.}$$

$\bar{H}$ only consists of $H_\phi$



(a) $\nabla \times \bar{H} = \bar{J}$

$$\nabla \times \bar{H} = -\frac{\partial H_\phi}{\partial z}\hat{a}_\rho + \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho H_\phi)\hat{a}_z.$$

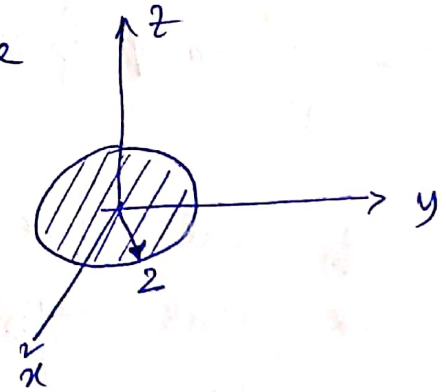
$H_\phi$ is independent of $z$.

$$= 0 + \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho \cdot 10^3 \rho^2)\hat{a}_z.$$

$$= \frac{1}{\rho} \cdot 3\rho^2 \cdot 10^3 \hat{a}_z = 3\rho\, \hat{a}_z \text{ kA/m}^2$$

(b) Current through the surface $0 < \rho < 2$, $0 < \phi < 2\pi$, $z = 0$

The differential surface ($d\bar{s}$) of the surface area shown $\Rightarrow d\bar{s} = \rho\, d\rho\, d\phi\, \hat{a}_z$



$$I = \int \bar{J} \cdot d\bar{s}$$

$$= \int (3 \times 10^3)\rho \cdot \hat{a}_z (\rho\, d\rho\, d\phi\, \hat{a}_z)$$

$$= \int_{\phi=0}^{\phi=2\pi} \int_{\rho=0}^{\rho=2} 3 \times 10^3 \rho^2\, d\rho\, d\phi = 3 \times 10^3 \int_0^2 \rho^2 d\rho \int_0^{2\pi} d\phi$$

$$= 3 \times 10^3 \left| \frac{\rho^3}{3} \right|_0^2 \cdot 2\pi = 8 \cdot 2\pi \times 10^3 \text{ A}$$

$$= 16\pi \text{ kA.}$$

(c). Ampere's law

$\bar{H} = 10^3 \rho^2 \hat{a}_\phi$ A/m.

$I_{enclosed} = \oint \bar{H} \cdot d\bar{l}$

Differential length


$d\bar{l} = 2 d\phi \, \hat{a}_\phi$.

$\rho = 2$.

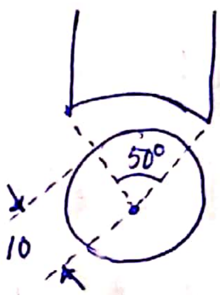
$$\oint \bar{H} \cdot d\bar{l} = \int_{\phi=0}^{\phi=2\pi} 10^3 \rho^2 \hat{a}_\phi \cdot 2 \, d\phi \, \hat{a}_\phi$$

$$= 2 \times 10^3 \times 4 \cdot \int_0^{2\pi} d\phi$$

$$= 8 \times 10^3 \times 2\pi = 16\pi \text{ kA}.$$

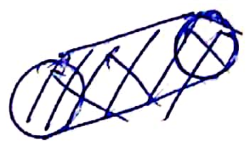
4.) Prob 7.36

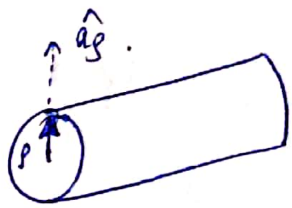
$\bar{H} = \frac{10^6}{\rho} \sin 2\phi \, \hat{a}_\rho$ H/m.




The differential surface ($d\bar{s}$) that the flux passes through air is along $\hat{a}_\rho$ axis

$d\bar{s} = \rho \, d\phi \, dz \, \hat{a}_\rho$.






Flux per pole passing through the airgap

$$\Psi = \int_S \bar{B} \cdot d\bar{s}$$

$$= \mu \int_S \frac{10^6}{\rho} \cdot \sin 2\phi \, \hat{a}_\rho \cdot \rho \, d\phi \, dz \, \hat{a}_\rho$$

$$\Psi = \mu \int_S 10^6 \cdot \sin 2\phi \cdot d\phi\, dz$$

$\mu$ (for air) $= \mu_0$

$$= \mu_0 \cdot \int_{\phi=0^\circ}^{\phi=50^\circ} \int_{z=0}^{z=20\,cm} 10^6 \sin 2\phi\, d\phi\, dz$$

$$= (4\pi \times 10^{-7}) \times 10^6 \int_{\phi=0^\circ}^{\phi=50^\circ} \sin 2\phi\, d\phi \int_{z=0}^{z=0.2} dz$$

$$= (4\pi \times 10^{-1}) \left[ \frac{-\cos 2\phi}{2} \right]_{\phi=0^\circ}^{\phi=50^\circ} \times (0.2)$$

$$= \frac{4\pi}{10} \cdot \frac{1}{2} (\cos 0^\circ - \cos 100^\circ) \times 0.2$$

$$= 0.14748 \text{ Wb.}$$

5a). Maxwell's Equations in differential form for static fields

$\nabla \cdot \bar{D} = \rho_v$ (Gauss's law).

$\nabla \cdot \bar{B} = 0$ (Free monopole doesnot exist)

$\nabla \times \bar{E} = 0$ (Conservative Electric field)

$\nabla \times \bar{H} = \bar{J}$ (Ampere's law).

5b) $\bar{D} = y^2 z \, \hat{a_x} + 2(x+1) yz \, \hat{a_y} - (x+1) z^2 \hat{a_z}$.

$$\nabla \cdot \bar{D} = \frac{\partial}{\partial x}(y^2 z) + \frac{\partial}{\partial y}\{2(x+1)yz\} + \frac{\partial}{\partial z}\{(-x-1)z^2\}$$

$$= 0 + 2(x+1)z - (x+1)2z.$$

$$= 0.$$

No monopoles exist for this field $(\bar{D})$.

$$\nabla \times \bar{D} = \begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ D_x & D_y & D_z \end{vmatrix}$$

$$= \left(\frac{\partial D_z}{\partial y} - \frac{\partial D_y}{\partial z}\right)\hat{a_x} + \left[\frac{\partial D_x}{\partial z} - \frac{\partial D_z}{\partial x}\right]\hat{a_y} + \left[\frac{\partial D_y}{\partial x} - \frac{\partial D_x}{\partial y}\right]\hat{a_z}.$$

$$= \{0 - 2(x+1)y\}\hat{a_x} + (y^2 - z^2)\hat{a_y} + (2yz - 2yz)\hat{a_z}.$$

$$= -2(x+1)y\,\hat{a_x} + (y^2 - z^2)\hat{a_y} \neq 0$$

$\bar{D}$ is not a conservative vector field.