1.) $\bar{E} = 3\hat{a}_y$ V/m

$\bar{B} = 5\hat{a}_z$ Wb/m^2

The particle starts from rest at the origin. (0,0,0).

Mass(m) = 1 kg

Charge(Q) = 1 C.

(a) Lorentz Force $F = Q(\bar{E} + \bar{u}\times\bar{B}) = ma = m\frac{du}{dt}$.

$$\bar{u}\times\bar{B} = \begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ u_x & u_y & u_z \\ 0 & 0 & 5 \end{vmatrix} = 5u_y\hat{a_x} - 5u_x\hat{a_y}.$$

$$Q(\bar{E}+\bar{u}\times\bar{B}) = m\frac{d\bar{u}}{dt}$$

$$\Rightarrow m\frac{d\bar{u}}{dt} = 1\left[3\hat{a_y} + 5u_y\hat{a_x} - 5u_x\hat{a_y}\right].$$

$$\Rightarrow \frac{d\bar{u}}{dt} = \cancel{3}\,5u_y\hat{a_x} + (3-5u_x)\hat{a_y}.$$

$$\therefore \frac{du_x}{dt} = 5u_y \rightarrow (1)$$

$$\frac{du_y}{dt} = 3-5u_x \rightarrow (2)$$

$$\frac{du_z}{dt} = 0 \rightarrow (3)$$

Taking second derivative of (2)

$$\frac{d^2u_y}{dt^2} = -5\frac{du_x}{dt}$$

$$\cancel{\Rightarrow \frac{du_x}{dt} = -\frac{1}{5}\frac{d^2u_y}{dt^2} = 5u_y}.$$

$$\frac{d^2 u_y}{dt^2} = -25 u_y.$$

$$\Rightarrow \frac{d^2 u_y}{dt^2} + 25 u_y = 0.$$

Expression of $u_y(t) \rightarrow u_y(t) = A\cos(5t) + B\,\overset{\sin}{\cancel{\cos}}(5t).$

~~At t=0, the particle starts from origin~~

At $t=0$, $u_y(0) = A\cos 0 + B\sin 0 = A$

$$u_y(0) = 0$$

$$\boxed{\therefore A = 0}$$

$$\therefore u_y(t) = B\sin 5t$$

$$\Rightarrow \frac{du_y}{dt} = 5B\cos 5t = 3 - 5u_x.$$

$$\therefore u_x = \frac{3}{5} - B\cos(5t).$$

At $t=0$, $u_x(0) = \frac{3}{5} - B\cos 0 = 0.$

$$\therefore B = 3/5$$

Hence, $u_x(t) = \frac{3}{5} - \frac{3}{5}\cos(5t) = \frac{3}{5}\left[1 - \cos(5t)\right]$ m/s.

$u_y(t) = \frac{3}{5}\sin(5t)$ m/s

From (3), $\frac{du_z}{dt} = 0 \Rightarrow u_z(t) = C.$

At $t=0$, $u_z(0) = C = 0 \Rightarrow u_z(t) = 0$ m/s.

At t=1,

$$\bar{u} = \frac{3}{5}(1-\cos 5t)\,\hat{a_x} + \frac{3}{5}\sin 5t\,\hat{a_y}$$

$$= 0.4298\,\hat{a_x} + -0.5753\,\hat{a_y}$$

$$\bar{u}(1) = (0.4298, -0.5753, 0)\ m/s$$

(b) $\frac{d\bar{r}}{dt} = \bar{u}$ $\qquad r = (x, y, z)$.

$$\frac{dx}{dt} = u_x = \frac{3}{5}(1-\cos 5t)$$

$$x(t) = \frac{3}{5}t - \frac{3}{25}\sin(5t) + A$$

At t=0 :- $x(0) = 0 - 0 + A = 0 \Rightarrow A = 0$.

$$\therefore x(t) = \frac{3}{5}t - \frac{3}{25}\sin(5t)\ m.$$

$$\frac{dy}{dt} = u_y = \frac{3}{5}\sin(5t) \Rightarrow y(t) = \frac{-3}{25}\cos(5t) + B$$

At t=0 $\quad y(0) = -\frac{3}{25} + B = 0 \Rightarrow B = 3/25$

$$y(t) = \frac{3}{25}(1-\cos(5t))\ m.$$

$$\frac{dz}{dt} = u_z = 0 \rightarrow u_z(t) = C$$

At t=0, $z(0) = C = 0 \Rightarrow z(t) = 0$.

At t=1:- $\bar{r} = (0.715, 0.0859, 0)\ m$

(c) Kinetic energy at t=1s $\Rightarrow \frac{1}{2}mu^2 = \frac{1}{2}\,1 \times (0.4298^2 + 0.5753^2)$

$$= 0.25784\ J.$$

2) $\bar{E} = 3\hat{a_y}$ $\qquad \bar{F} = Q(\bar{E} + \cancel{\bar{u} \times \bar{B}}^{0}) = m\frac{d\bar{u}}{dt}$

$\bar{B} = 0$

$r_0 = (0,0,0)$ $\qquad \therefore \frac{d\bar{u}}{dt} = \frac{Q}{m}\bar{E} = 3\hat{a_y}$

$u_0 = (0,0,0)$

$Q = 1C$ $\qquad \therefore \frac{du_x}{dt} = 0 \Rightarrow u_x(t) = A \rightarrow (1)$

$m = 1kg$

(a) $\qquad \frac{du_y}{dt} = 3 \Rightarrow u_y(t) = 3t + B \rightarrow (2)$

$\frac{du_z}{dt} = 0 \Rightarrow u_z(t) = C \rightarrow (3)$

From (1)

At $t=0$, $u_x(0) = A = 0 \Rightarrow u_x(t) = 0$.

from (2)

At $t=0$, $u_y(0) = 3(0) + B = 0 \Rightarrow u_y(t) = 3t$ m/s.

from (3).

At $t=0$, $u_z(0) = C = 0 \rightarrow u_z(t) = 0$.

At $t = 1s$:- $\boxed{\bar{u}(1) = 3\hat{a_y} \text{ m/s}}$

b) $\frac{dx}{dt} = u_x = 0 \Rightarrow x(t) = A$.

At $t=0$, $x(0) = A = 0 \Rightarrow x(t) = 0$

$\frac{dy}{dt} = u_y = 3t \Rightarrow y(t) = \frac{3}{2}t^2 + B$

At $t=0$, $y(0) = \frac{3}{2}\cdot 0 + B \Rightarrow y(t) = \frac{3}{2}t^2$ m.

$\frac{dz}{dt} = u_z = 0 \Rightarrow z(t) = c.$

At $t=0$, $z(0) = c = 0 \Rightarrow z(t) = c.$

At $t=0$:- $z(0) = c = 0 \rightarrow z(t) = 0.$

At $t = 1s$:- $\boxed{\begin{array}{l} \bar{r}(1) = 1.5\hat{a}_y \ m \\ \bar{r}(1) = (0, 1.5, 0) \ m. \end{array}}$

(c) Kinetic energy at $t = 1s$,

$$KE = \frac{1}{2}mu^2 = \frac{1}{2} \cdot 1.9 = 4.5 \ J$$

3) $E = 0$

$\bar{B} = 5\hat{a}_z \ Wb/m.$

$r_0 = (0,0,0)$

$u_0 = (0,0,0)$

$$\bar{F} = Q(\overset{0}{\bar{E}} + \bar{u} \times \bar{B}) = m\frac{d\bar{u}}{dt}$$

Particle is initially at rest,

$\bar{B}$ won't exert force on the particle.

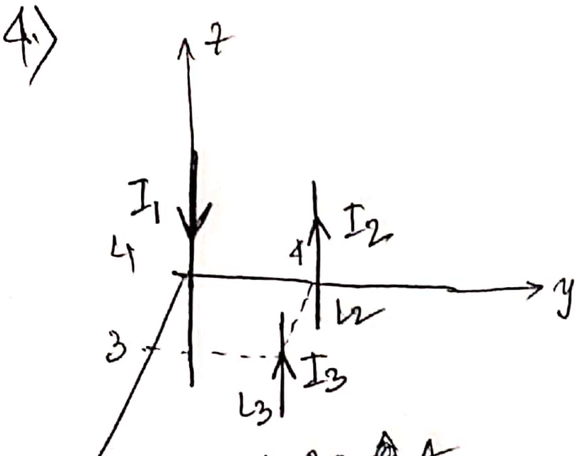
Thus, the particle will never move.

(a) $\bar{u}(1) = \bar{u}(0) = 0$

$\bar{r}(1) = \bar{r}(0) = (0,0,0).$

At $t = 1s$, $KE = 0.$

4.)



$I_1 = +100\ A$

$I_2 = 200\ A$

$I_3 = 300A$.

Force per unit length

(a) L_2 due to L_1

$$\bar{F}_{21} = I_2 \bar{L}_2 \times \bar{B}_1 \qquad \bar{B}_1 = \frac{-\mu_0 I_1}{2\pi\rho}\hat{a}_\phi.$$

$$= I_2 \bar{L}_2 \times \left(-\frac{\mu_0 I_1}{2\pi\rho}\right)\hat{a}_\phi \qquad \bar{L}_2 = L_2\hat{a}_z.$$

$$= \frac{\mu_0 I_1 I_2}{2\pi\rho}(-\hat{a}_z)\hat{a}_\phi . L_y$$

$$= \frac{4\pi\times10^{-7}\times100\times200}{2\pi\times4} L_2 \hat{a}_\rho$$

$$= 10^{-3} L_2 \hat{a}_\rho.$$

$$\boxed{\frac{\bar{F}_{21}}{L_2} = 1\ \hat{a}_\rho\ mN/m}$$

It is a repulsive force.

(b) L_3 due to L_1

$$\bar{F}_{31} = I_3 \bar{L}_3 \times \bar{B}_1 \qquad B_1 = \frac{-\mu_0 I_1}{2\pi\rho}\hat{a}_\phi.$$

$$\bar{L}_3 = L_3\hat{a}_z$$

$$= \frac{I_3 . \mu_0 I_1}{2\pi\rho} L_3 \cdot (-\hat{a}_z)\hat{a}_\phi$$

$$= \frac{300\times100\times4\pi\times10^{-7}}{2\pi\rho} L_3 \hat{a}_\rho$$

$$\rho = \sqrt{3^2+4^2} = 5.$$

$$\overline{F_{31}} = \frac{300 \times 100 \times 4\pi \times 10^{-7}}{2\pi \times 5} L_3 \hat{a_\rho}$$

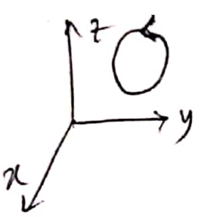
$$\frac{\overline{F_{31}}}{L_3} = 1.2 \; \hat{a_\rho} \quad mN/m$$

$$= 1.2\left(\cos\phi \; \hat{a_x} + \sin\phi \; \hat{a_y}\right) \; mN/m.$$

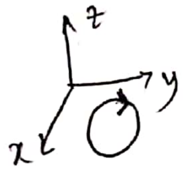
$$\phi = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{4}{3} = 53.19^\circ.$$

$$= 0.72\hat{a_x} + 0.96\hat{a_y} \quad mN/m.$$

5.)

a) 

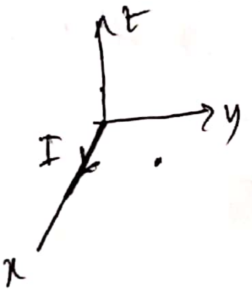
use right hand rule, $\bar{m}$ is in $+\hat{a_x}$ direction.

b) 

Moment due to current loop ($\bar{m}$) is in $-\hat{a_z}$ direction.

$\bar{B}$ is along $\hat{a_x}$ direction.

$\gamma = \bar{m} \times \bar{B} = -\hat{a_z} \times \hat{a_x} = -\hat{a_y}$ direction.

c) 

Lets find out $\bar{B}$ due to filamental current along $+\hat{x}$ direction.

In this case, $d\bar{l} = dx\,\hat{a_x}$

$\bar{R} = \rho\hat{a_\rho} - x\hat{a_x}$ (But ρ is along y direction)

$= y\hat{a_y} - x\hat{a_x}.$

$d\bar{l} \times \bar{R} = dx\,\hat{a_x} \times (y\hat{a_y} - x\hat{a_x}) = y\,dx\,\hat{a_z}.$

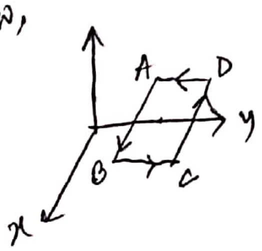
$$\therefore B = \int_{-\infty}^{\infty} \frac{I y\, dx\, \hat{a_z}}{4\pi (x^2+y^2)^{3/2}}$$

$$= \frac{I}{4\pi y} \left. \frac{x}{\sqrt{x^2+y^2}} \right|_{x=-\infty}^{x=\infty}$$

$$= \frac{I}{2\pi y} \hat{a_z}.$$

$\therefore \bar{B}$ is along $\hat{a_z}$ direction

Now,



$\overline{AD}$ is along $-\hat{a_y}$

$\overline{BC}$ along $\hat{a_y}$

$\overline{AB}$ along $\hat{a_x}$

$\overline{CD}$ along $-\hat{a_x}$.

$\vec{F_{BC}} = \hat{a_y} \times \hat{a_z}$

$= \hat{a_x}.$

~~For AB & CD, the force is zero since~~

$\bar{F_{AB}} \rightarrow \bar{l_{AB}} \times \bar{B} \rightarrow \hat{a_x} \times \hat{a_z} \rightarrow -\hat{a_y}$

$F_{CD} = -\hat{a_x} \times \hat{a_z} = \hat{a_y}$

$\bar{F_{AD}} \rightarrow -\hat{a_y} \times \hat{a_z} = -\hat{a_x}$

6a) Magnet at origin produces $\bar{B} = -0.5\ Wb/m^2$ at $(10,0,0)$

$(10,0,0)$ in (x,y,z) coordinates $\equiv (10,\pi/2,0)$ in (r,θ,ϕ) coordinates.

Magnetic flux density due to dipole

$$\bar{B} = \frac{\mu_0 m}{4\pi r^3}\left(2\cos\theta\,\hat{a}_r + \sin\theta\,\hat{a}_\theta\right) = -0.5\ mWb/m^2.$$

$$\bar{B}\Big|_{(10,\pi/2,0)} = \frac{\mu_0 m}{4\pi \times 10^3}\left(2\cos\pi/2\,\hat{a}_r + \sin\pi/2\,\hat{a}_\theta\right)$$

$$= \frac{4\pi\times10^{-7} m}{4\pi\times10^3}\cdot\hat{a}_\theta.$$

$$m = \frac{0.5\times10^{-3}}{10^{-7}}\times10^3 = 5\times10^6.$$

at $\bar{r} = (3,4,0)m \Rightarrow r = \sqrt{3^2+4^2} = 5,\ \theta = \pi/2.$

$$\bar{B} = \frac{\mu_0\, 5\times10^6}{4\pi\times5^3}\sin\pi/2\,\hat{a}_\theta = \frac{10^6}{25}\times10^{-7}\,\hat{a}_\theta\ Wb/m^2$$

$$= 4\hat{a}_\theta\ mWb/m^2.$$

(b) $\chi_m = \mu = 4.6\mu_0\ H/m,\ \bar{B} = 5x\hat{a}_z\ Wb/m^2.$

$$\mu_r = 4.6 = (1+\chi_m)$$

$$\Rightarrow \chi_m = 4.6-1 = 3.6.$$

(c) $$\bar{H} = B/\mu = \frac{5x\hat{a}_z}{4.6\mu_0} = 864972.5168x\,\hat{a}_z\ A/m$$

$$= 864.97x\,\hat{a}_z\ kA/m.$$

7)

①	②
$x<0$	$x>0.$
$\mu_1=\mu_0$	$\mu_2=50\mu_0$
$\bar{B}_1$	$\bar{H}_2=?$

Boundary Conditions

1) $H_{1t}=H_{2t}$

2) $B_{1n}=B_{2n}$

$\bar{B}=\mu\bar{H}$

$\bar{B}_1=40\hat{a}_x-30\hat{a}_y+10\hat{a}_z$ mWb/m^2

$$\bar{B}_{1n}=40\times10^{-3}\hat{a}_x=B_{2n}.$$

$$H_{2n}=\frac{B_{2n}}{\mu_2}=\frac{40\times10^{-3}}{50\mu_0}\hat{a}_x=636.619\,\hat{a}_x\ \text{A/m}.$$

$$\bar{B}_{1t}=(-30\hat{a}_y+10\hat{a}_z)\times10^{-3}=\mu_1\bar{H}_{1t}$$

$$\bar{H}_{1t}=\frac{(-30\hat{a}_y+10\hat{a}_z)\times10^{-3}}{\mu_0}=-23.87\hat{a}_y+7.957\hat{a}_z\ \text{kA/m}$$

$$\bar{H}_{1t}=\bar{H}_{2t}$$

$$\therefore\ \bar{H}_2=(0.6366\,\hat{a}_x-23.87\hat{a}_y+7.957\hat{a}_z)\ \text{kA/m}.$$

b) Energy Density in region 2.

$$W_m=\frac{1}{2}\mu_2H^2=\frac{1}{2}\times50\mu_0\times\{(0.6366\times10^3)^2+(23.87\times10^3)^2+(7.957\times10^3)^2\}$$

$$=19.90\ \text{kJ/m}^3$$

8.) Inductance for Solenoid

$$L = \frac{\mu_0 N^2 S}{l}.$$

$l = 10$ cm.

$r = 1$ cm

$N = 450.$

$S = \pi r^2$

$$= \frac{\mu_0 N^2 \pi r^2}{l}$$

$$= \frac{4\pi \times 10^{-7} \times 450^2 \times \pi \times (0.01)^2}{0.1}$$

$= 799.43$ ~~mH~~ μH

(b) For coaxial transmission line

$$L = \frac{\mu_0 l}{2\pi} \ln b/a$$

$b = 6$ mm

$a = 2.5$ mm.

$\mu_r = 1.$

$$L/l = \frac{\mu_0}{2\pi} \ln b/a$$

$$= \frac{4\pi \times 10^{-7}}{2\pi} \ln \frac{6}{2.5} = 175.09 \text{ nH/m}.$$