1) $w = 6\,mm$, $d = 0.25\,mm$, $t = 25\,mm$, $\sigma_c = 5.5 \times 10^{7}\ S/m$, $\sigma = 3.5 \times 10^{-3}\ S/m$, $\epsilon = 25\,pF/m$, $f = 750\ MHz$, $\mu = \mu_c = \mu_0 = 4\pi \times 10^{-7}\ H/m$

Resistance $R(\Omega/m) = \dfrac{2}{w\delta\sigma_c} = \dfrac{2\sqrt{\pi f \mu_c \sigma_c}}{w\sigma_c} = \dfrac{2\sqrt{\pi f \mu_c}}{w\sqrt{\sigma_c}} = 2.445\ \Omega/m$

(for $\delta << t$)

$\delta \to$ Skin depth

Since, $\delta = \dfrac{1}{\sqrt{\pi f \mu_c \sigma_c}} = 2.47 \times 10^{-6}\,m$

$\delta << t$

Inductance $L(H/m) = \dfrac{\mu d}{w}$

Conductance $G(S/m) = \dfrac{\sigma w}{d}$.

Capacitance $C(F/m) = \dfrac{\epsilon w}{d}$

Inductance $L = \dfrac{\mu d}{w} = \dfrac{4\pi \times 10^{-7} \times 0.25 \times 10^{-3}}{6 \times 10^{-3}} = 5.23 \times 10^{-8}\ H/m$.
$= 52.3\ nH/m$.

Conductance $G = \dfrac{\sigma w}{d} = \dfrac{3.5 \times 10^{-3} \times 6 \times 10^{-3}}{0.25 \times 10^{-3}} = 0.084\ S/m$
$= 84\ mS/metre$.

Capacitance $C = \dfrac{\epsilon w}{d} = \dfrac{25 \times 10^{-12} \times 6 \times 10^{-3}}{0.25 \times 10^{-3}} = 600 \times 10^{-12}\ F/m$
$= 600\ pF/m$.

2a) Propagation constant $\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$

$$= \sqrt{(2.445 + j \times 2\pi \times 750 \times 10^{6} \times 5.23 \times 10^{-8})(84 \times 10^{-3} + j 2\pi \times 750 \times 10^{6} \times 600 \times 10^{-12})}$$

$$= \sqrt{(2.445 + j\,246.457)(84 \times 10^{-3} + j\,2.827)}$$

$$\gamma = (697.075\angle 177.729^\circ)^{1/2}$$

$$= \sqrt{697.075}\ \angle\left(\frac{177.729^\circ}{2}\right)$$

$$= 26.4\angle 88.86^\circ = 0.5252 + j\,26.394\ \ 1/m.$$

2(b) The characteristic impedance $(Z_0) = \sqrt{\dfrac{R + j\omega L}{G + j\omega C}}$

$$Z_0 = \sqrt{\frac{2.445 + j\,246.457}{84\times10^{-3} + j\,2.827}}$$

$$= \sqrt{(87.145\angle 1.133^\circ)}$$

$$= \sqrt{87.145}\ \angle\frac{1.133^\circ}{2}$$

$$= 9.33\angle 0.5665^\circ = 9.3295 + j\,0.09224\ \Omega$$

2c) When real power loss in lines is neglected, line resistance can be considered as zero $(R=0)$.

When dielectrics are considered to be ideal i.e. they act as a pure capacitance and no real power losses happen ~~due to eddy current [illegible]~~ due to flow of current through them, conductance is taken as zero $(G=0)$.

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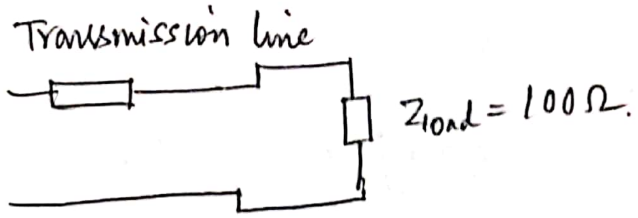
3.)



Voltage reflection coefficient (Γ_L) when transmission line of characteristic impedance (Z_0) is terminated with a load of impedance (Z_L) is given as—

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100 - (9.3295 + j0.09224)}{100 + (9.3295 + j0.09224)}$$

$$= 0.8293 - j1.5948 \times 10^{-3}$$

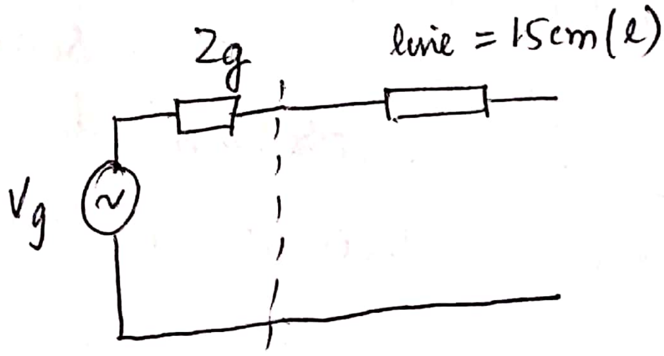
$$= 0.8293 \angle -0.106^{\circ}$$

Standing wave ratio (SWR) $= \dfrac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$

$$= \frac{1 + 0.8293}{1 - 0.8293}$$

$$= 10.716$$

4.)



4.) $Z_{in} = Z_0 \left[\frac{Z_L + Z_0 \tanh(\gamma L)}{Z_0 + Z_L \tanh(\gamma L)} \right].$

$Z_L = 100\,\Omega$

$Z_0 = (9.395 + j0.09224)\,\Omega.$

$\tanh \gamma L = \tanh(\alpha l + j\beta l)$

$= \frac{\tanh(\alpha L) + j\tan(\beta L)}{1 + j\tanh(\alpha L)\tan(\beta L)}.$

Length of line $(l) = 15$ cm.

$\therefore \gamma l = (0.5252 + j26.394) \times 0.15$

$= 0.078 + j3.9591$ radians

$= \alpha l + j\beta l.$

$\therefore \tanh(\gamma L) = \frac{\tanh(0.078) + j\tan(3.9591)}{1 + j\tanh(0.078)\tan(3.9591)}$

$= \frac{0.0778 + j1.0663}{1 + j0.083} = 0.165 + j1.052$

$Z_{in} = (9.395 + j0.09224) \left[\frac{100 + (9.395 + j0.09224)(0.165 + j1.052)}{(9.395 + j0.09224) + j100(0.165 + j1.052)} \right]$

$= \frac{(9.395 + j0.09224)(101.453 + j9.898)}{(25.895 + j105.292)}$

$= (3.0139 - j8.3025)\,\Omega = 8.83 \angle -1.222 \text{ rad } \Omega$

4b)

$$V_0^+ = \left(\frac{Z_{in}}{Z_{in}+Z_g}\right)\frac{V_g}{e^{\gamma l}+\Gamma_L e^{-\gamma l}}.$$

$$e^{\gamma l} = e^{(0.5252+j26.394)\times 0.15}$$

$$= e^{0.07878}\, e^{j3.9591}$$

$$= -0.7401-j0.789.$$

$$e^{-\gamma l} = e^{(-0.5252-j26.394)\times 0.15}$$

$$= e^{-0.07878}\, e^{-j3.9591}$$

$$= -0.6322+j0.6742.$$

$$\Gamma_L = 0.8293-j1.5448\times10^{-3}.$$

$$V_0^+ = \frac{(3.0139-j8.3025)\times 12}{(3.0139-j8.3025+10)\times\left[(-0.7401-j0.789)+(0.8293-j1.5448\times10^{-3})\times(-0.6322+j0.6742)\right]}$$

$$= \frac{(3.0139-j8.3025)\times12}{(13.0139-j8.3025)(-1.2633-j0.228)}$$

$$= -3.5926+j3.96 = 5.346\angle 2.307\text{ rad Volts.}$$

5a) $V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$ $\qquad \Gamma_L = \frac{V_0^-}{V_0^+}$

$= V_0^+ (e^{-\gamma z} + \Gamma_L e^{\gamma z})$

$= |V_0^+| e^{j\phi_g} \left[e^{-\alpha z - j\beta z} + \Gamma_L e^{+\alpha z + j\beta z} \right]$ $\phi_g \Rightarrow$ phase angle of source.

$\gamma = 0.5252 + j26.394$

$= \alpha + j\beta$

$\therefore \alpha = 0.5252$

$\beta = 26.394$

$\phi_g = 0°$.

$V_0^+ = 5.346 \angle 2.307$ Volts.

$\Gamma_L = 0.8293 - j1.5448 \times 10^{-3}$

$$V_s(z) = 5.346 \angle 2.307 \left[e^{(-0.5252 - j26.394)z} + (0.8293 - j1.5448 \times 10^{-3}) e^{+(0.5252 + j26.394)z} \right]$$

$$= 5.346\, e^{j2.307} \left[e^{(-0.5252 - j26.394)z} + 0.8293\, e^{-j0.00186} e^{(0.5252 + j26.394)z} \right]$$

$$= 5.346\, e^{(-0.5252 - j26.394)z + j2.307} + 4.433\, e^{(0.5252 + j26.394)z - j0.00186}$$

Volts

5 b)

$$V(z,t) = Re\left[V_s(z)\, e^{j\omega t}\right]$$

$$= Re\left[V_s(z)\, e^{j2\pi \times 750 \times 10^6 t}\right]$$

$$= Re\left[V_s(z)\, e^{j4.71 \times 10^9 t}\right].$$

radians

$$V(z,t) = 5.346\, e^{-0.5252z} \cos\left(4.71\times10^9 t - 26.394z + 2.307\right)$$

radians

$$+\, 4.433\, e^{0.5252z} \cos\left(4.71\times10^9 t + 26.394z - 0.00186\right)$$

Volts