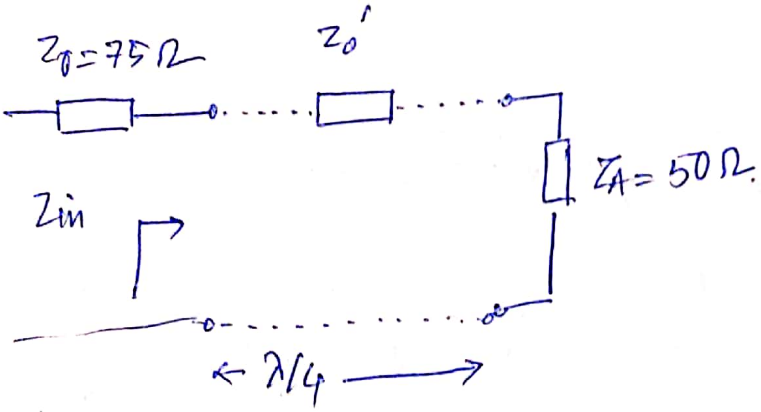
EE 331 Homework 4

1.)

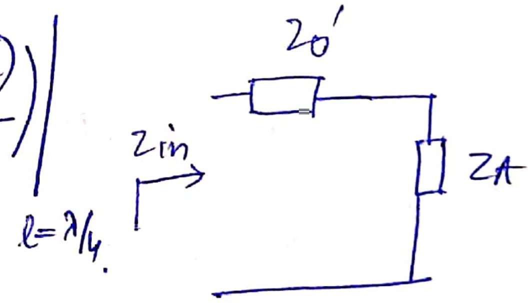


Let, the characteristic impedance of the transmission line connecting the 75 Ω transmission line and the antenna (of impedance $Z_A = 50\Omega$) be Z_0'. The length of this line is ($l = \lambda/4$). → Quarter-wave length transformer.

The purpose of the quarter-wave length transformer is to match the antenna load with that of the transmission line of ($Z_0 = 75\Omega$).

Hence, the input impedance as seen by the transmission line should also be ($Z_{in} = 75\Omega$)

$$Z_{in}(l = \lambda/4) = Z_0' \left(\frac{Z_A + jZ_0 \tan(\beta l)}{Z_0 + jZ_A \tan(\beta l)} \right)$$



$$\beta l \Big|_{l=\lambda/4} = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$$

$\tan(\pi/2) \to \infty$.

$$\therefore Z_{in}(l = \lambda/4) = Z_0' \frac{Z_A + jZ_0 \tan(\beta l)}{Z_0 + jZ_A \tan(\beta l)}$$

$$= \frac{(Z_0')^2}{Z_A}$$

$$Z_{in} = \frac{Z_0'^2}{Z_A}$$

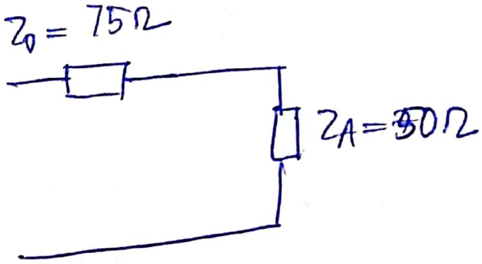
$$\Rightarrow Z_0' = \sqrt{Z_{in} Z_A} = \sqrt{75 \times 50} = 61.23\,\Omega \text{ (Ans)}$$

$$\text{Length of the line} = \frac{\lambda}{4} = \frac{u}{4f} = \frac{3\times10^8}{4\times(97.1\times10^6)} = 0.772\,m$$

$$= 77.2\,cm \text{ (Ans).}$$

b) Power delivered to the antenna without matched line is expressed as →

$$P_{av} = \frac{1}{2}\frac{|V_0^+|^2}{Z_0}\left[1 - |\Gamma|^2\right].$$



$$\Gamma = \frac{Z_A - Z_0}{Z_A + Z_0} = \frac{50-75}{50+75}$$

$$= \frac{-25}{125} = -\frac{1}{5}.$$

$$P_{av} = \frac{1}{2}\frac{|V_0^+|^2}{Z_0}\left\{1 - \left(\tfrac{1}{5}\right)^2\right\}.$$

Power delivered to the antenna with matched line $(\Gamma = 0)$

$$\therefore P_{av}' = \frac{1}{2}\frac{|V_0^+|^2}{Z_0}.$$

$$\therefore \frac{P_{av}}{P_{av}'} = \frac{\left\{\tfrac{1}{2}|V_0^+|^2/Z_0\right\}}{\left\{\tfrac{1}{2}|V_0^+|^2/Z_0\right\}} \cdot \frac{1-\tfrac{1}{25}}{1} = \frac{24}{25} \text{ (Ans)}$$

Chdory

2) Z_0 (Characteristic impedance) = 50Ω.

Z_L (load impedance) = $(40-j25)\,\Omega$.

$$Z_L\,(\text{normalized}) = \frac{40-j25}{50} = 0.8-j0.5.$$

Z_L is given by the point P.

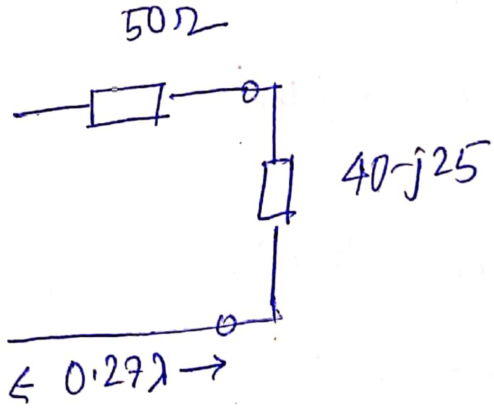
$$\left.\begin{aligned} |\Gamma_L| &= \frac{OP}{OQ} = 0.282. \\ \angle\Gamma_L &= -96^\circ. \end{aligned}\right\}\ \text{From Smith's chart.}$$

a) $\Gamma_L = 0.282\angle{-96^\circ}$

b) Put the distance of 'OP' on SWR line below, you will get

$SWR(S) = 1.8$.

c)



To find Z_{in}, draw the constant VSWR circle with OP as the radius and rotate the point P (in the direction of generator i.e. clockwise by an additional 0.27λ).

Current position of OP ⇒ 0.382λ.

$0.382\lambda + 0.27\lambda = 0.652\lambda$.

∴ Position of Z_{in} ⇒ $= (0.652\lambda - 0.5\lambda) = 0.152\lambda$ (in the upper half of chart)

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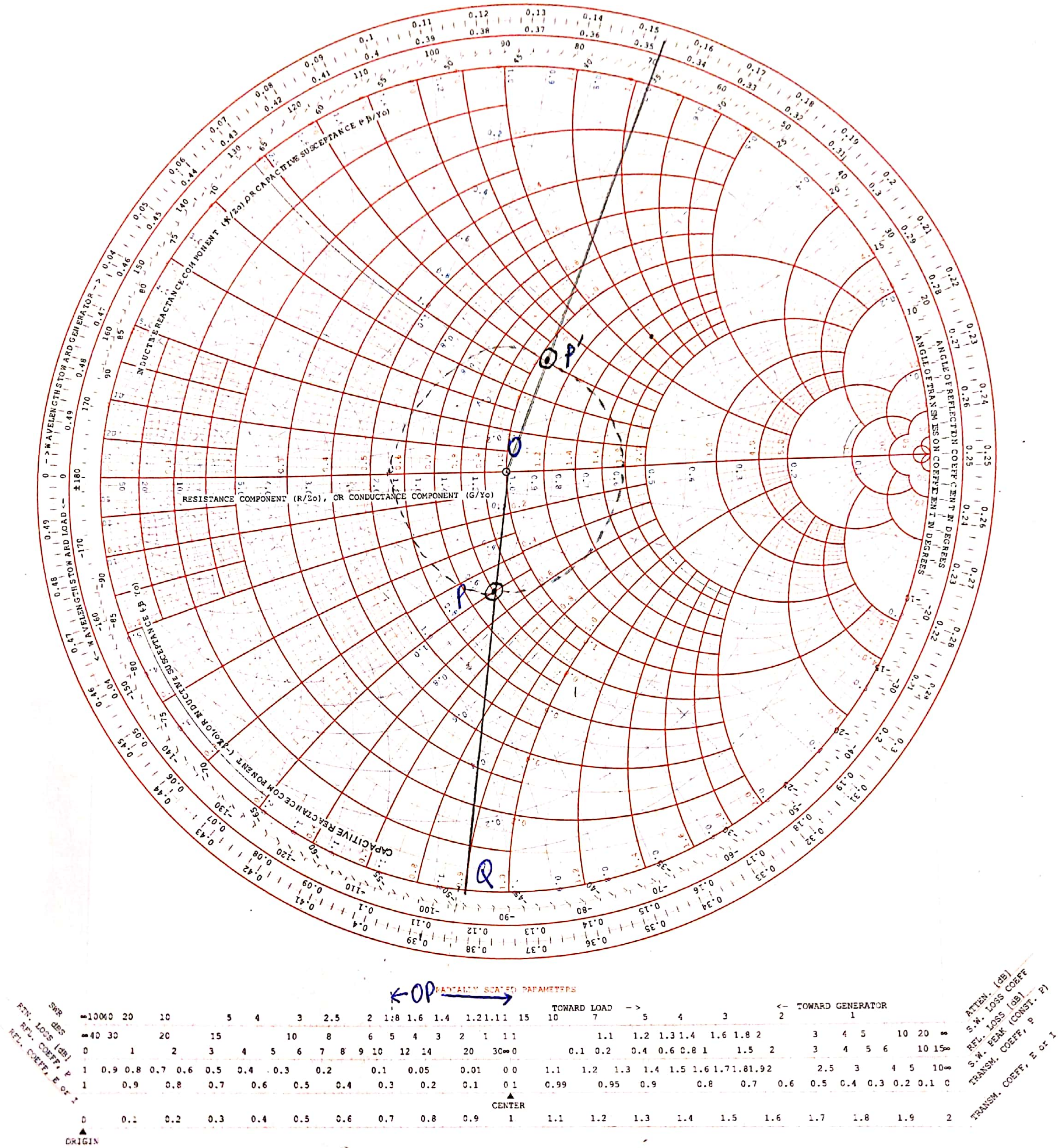
NAME Problem 2 TITLE DWG. NO.

DATE

SMITH CHART ENGS 120 COLOR BY J. COLVIN, UNIVERSITY OF FLORIDA, 1997

NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES



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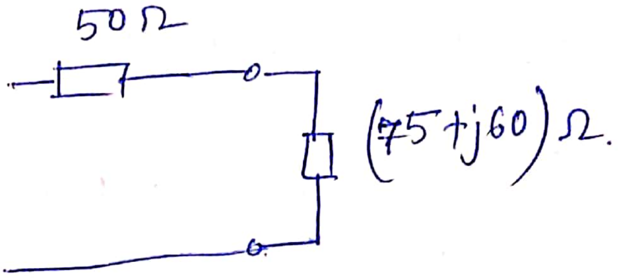
The point (Z_{in}) is given by P'.

$$Z_{in} \cong (1 + j0.6)$$
$$= (1+j0.6) \times 50 = (50 + j30)\,\Omega$$

3) Prob 11.43

a)



$$Z_L = \frac{75+j60}{50} = (1.5 + j1.2)$$

$$|\Gamma_L| = \frac{OP}{OQ} = 0.487$$
$$\angle \Gamma_L = 41^\circ = \angle AOQ$$

From Smith chart.

$$\Gamma_L = 0.487 \angle 41^\circ.$$

b) SWR is shown below.

$$S = 2.75$$

c) Draw the constant VSWR circle with OP as radius. and rotate 0.2λ towards generator.

Z_{in} is shown by P'.

$$Z_{in} = (0.53 - j0.65)\,50 = (26.5 - j32.5)\,\Omega.$$

NAME Problem 3 TITLE DWG. NO.

SMITH CHART ENGS 120 DATE

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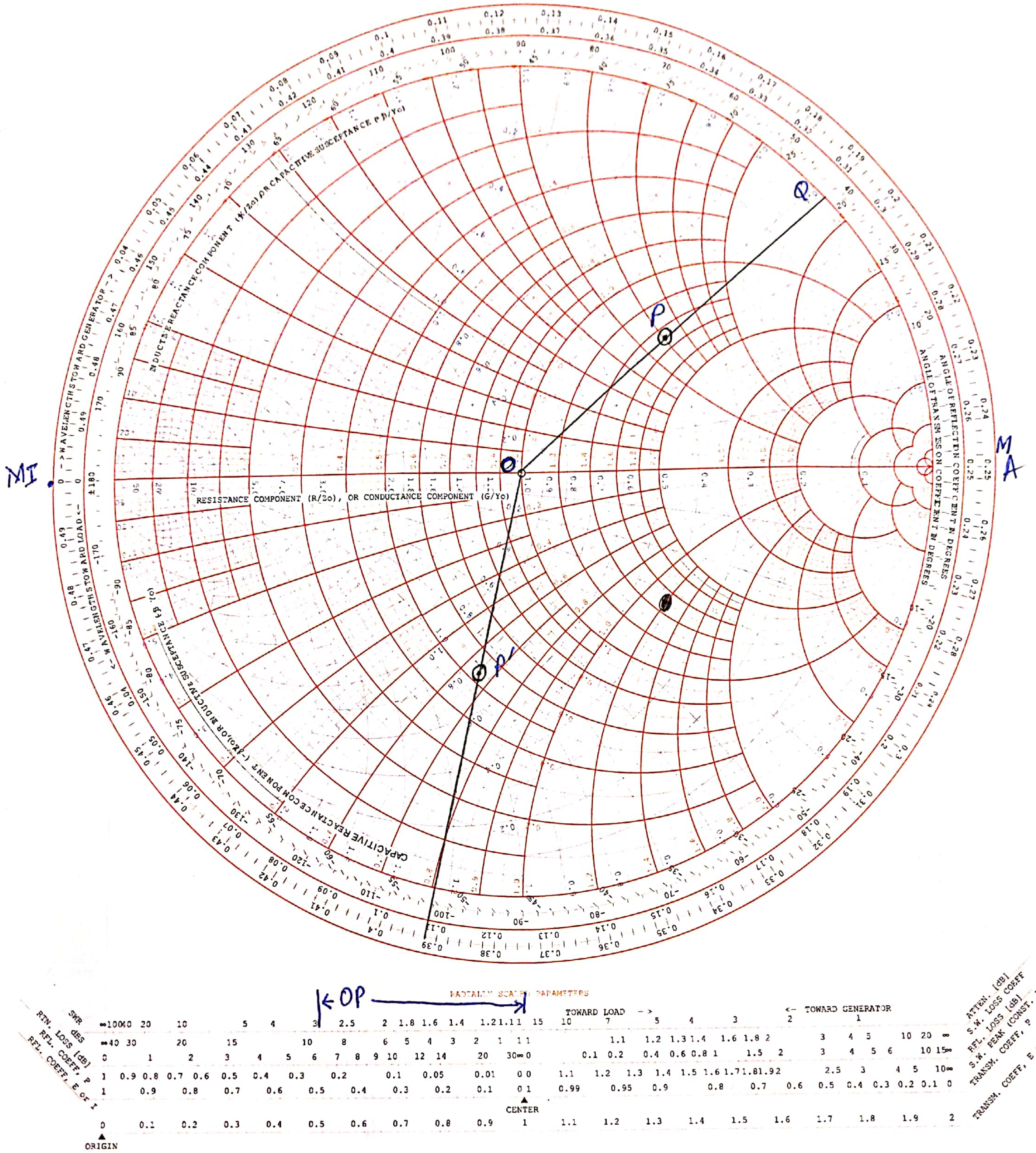
NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES



d) Minimum voltage occurs when the load is shorted $(r = jx = 0)$.
The point of minimum voltage is shown by the extreme left point in the chart (MI).

The distance from OQ to MI going towards generator

$$= (0.5\lambda - 0.192\lambda) = 0.308\lambda.$$

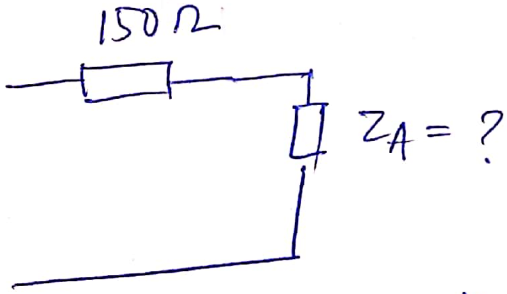
e) Maximum voltage occurs when the load is opened $(r = jx \to \infty)$
The point of maximum voltage is shown by the extreme right point in the chart (MA).
The distance from OQ to MA going towards generator

$$= (0.25\lambda - 0.192\lambda) = 0.058\lambda.$$

4.) <u>Prob 11.50</u>

a)

$s = 2.6.$



The voltage maxima and minima are located $\lambda/2$ distance apart.

$\therefore \lambda/2 = 120\text{ cm} \Rightarrow \lambda = 240\text{ cm}.$

$$f = \frac{u}{\lambda} = \frac{3\times10^8}{2.4} = 125\text{ MHz}$$

NAME Problem 4

TITLE

DWG. NO.

DATE

SMITH CHART ENGS 120

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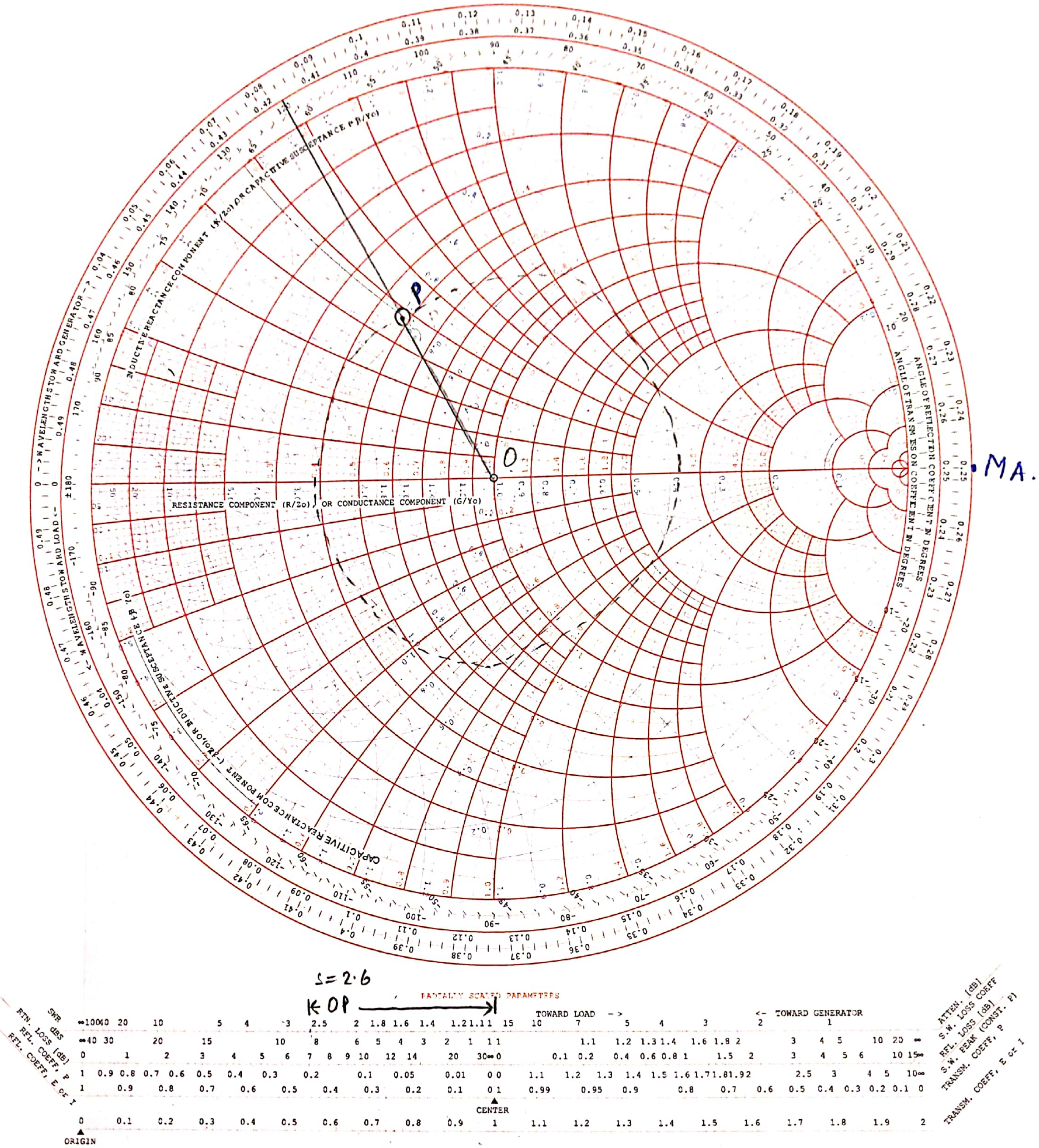
NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES



b). The last maximum is 40 cm from the antenna

$$40\text{ cm} = \frac{40}{240}\lambda = 0.167\lambda.$$

Draw the constant VSWR ($s = 2.6$) circle and the antenna is located at 0.167λ 'towards load' from the maximum voltage (extreme right) point given by M_A

location of antenna $= (0.25\lambda + 0.167\lambda)$
$= 0.417\lambda$ (upper half plane of the Smith's chart)

location of antenna → OP.

$$Z_A = (0.48 + j0.48)$$

$$= (0.48 + j0.48)\,150$$

$$= (72 + j72)\ \Omega. \quad \text{(Ans).}$$