EE 331 Homework #6

1) a) $\bar{A} = 4\hat{a}_x - 2\hat{a}_y + 6\hat{a}_z$

$\bar{B} = 12\hat{a}_x + 18\hat{a}_y - 8\hat{a}_z$

$\bar{A}.\bar{B} = (4\hat{a}_x - 2\hat{a}_y + 6\hat{a}_z).(12\hat{a}_x + 18\hat{a}_y - 8\hat{a}_z)$

$= 4.12 + (-2).18 + 6(-8)$

$= -36.$

$\hat{a}_x.\hat{a}_x = 1$
$\hat{a}_y.\hat{a}_y = 1$
$\hat{a}_z.\hat{a}_z = 1$
$\hat{a}_x.\hat{a}_y = 0$
$\hat{a}_y.\hat{a}_z = 0$
$\hat{a}_x.\hat{a}_z = 0$

b) $\bar{A} \times \bar{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

where, $\bar{A} = A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z$

$\bar{B} = B_x\hat{a}_x + B_y\hat{a}_y + B_z\hat{a}_z$

where $\hat{a}_x, \hat{a}_y, \hat{a}_z$ are unit vectors along x, y and z directions respectively.

$\therefore \bar{A} \times \bar{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 4 & -2 & 6 \\ 12 & 18 & -8 \end{vmatrix}$

$= \left[(-2)(-8) - 18.6\right]\hat{a}_x - \left[4(-8) - 12.6\right]\hat{a}_y + \left[4.18 + 12.2\right]\hat{a}_z$

$= -92\hat{a}_x + 104\hat{a}_y + 96\hat{a}_z.$

c) Unit vector of $\bar{A}$ be $\hat{a}$.

$$\bar{A} = |A|\,\hat{a}.$$

$$\therefore \hat{a} = \frac{\bar{A}}{|A|} = \frac{4a_x - 2a_y + 6a_z}{\sqrt{4^2+2^2+6^2}}$$

$$= 0.534a_x - 0.26726a_y + 0.8017a_z.$$

2.) $\bar{A} = 4a_x - 2a_y + 6a_z$

a) $\bar{B} = 12a_x + 18a_y - 8a_z.$

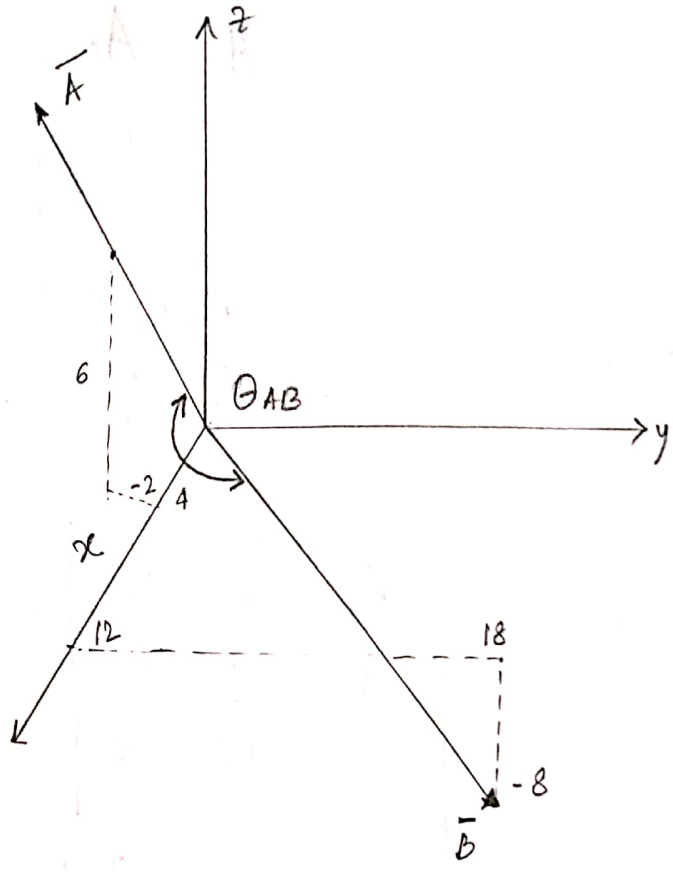
$$\bar{A}.\bar{B} = |\bar{A}||\bar{B}|\cos\theta_{AB}$$

$$\cos\theta_{AB} = \frac{\bar{A}.\bar{B}}{|\bar{A}||\bar{B}|}$$

$$= \frac{-36}{\sqrt{4^2+2^2+6^2}.\sqrt{12^2+18^2+8^2}}$$

$$= -0.2085$$

$$\theta_{AB} = 102.03^0$$



b) Scalar projection of $\bar{A}$ along $\bar{B}$ is given by

$$P_B(\bar{A}) = \frac{\bar{A}\cdot\bar{B}}{|\bar{B}|}$$

$$= \frac{-36}{\sqrt{12^2+18^2+8^2}} = -1.5608$$

c) Vector projection of $\bar{B}$ along $\bar{A}$ is given by

$$\text{proj}_{\bar{A}}(\bar{B}) = \frac{\bar{A}\cdot\bar{B}}{|\bar{A}|}\cdot\frac{\bar{A}}{|\bar{A}|} = \frac{\bar{A}\cdot\bar{B}}{|\bar{A}|^2}\cdot\bar{A}$$

$$= \frac{-36}{56}\left(4\hat{a_x} - 2\hat{a_y} + 6\hat{a_z}\right)$$

$$= -2.5714\hat{a_x} + 1.2857\hat{a_y} - 3.8571\hat{a_z}.$$

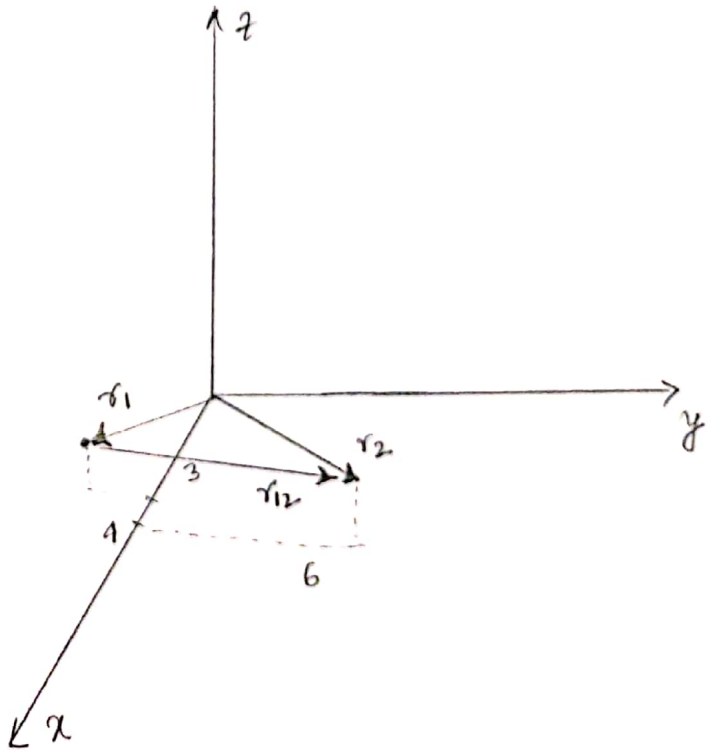
3.) Position vectors

a) $\vec{r_1} = (3, -2, 1)$ m

$\vec{r_2} = (4, 6, 2)$ m.

b) $\vec{r_{12}} + \vec{r_1} = \vec{r_2}$

$\vec{r_{12}} = \vec{r_2} - \vec{r_1} = (1, 8, 1)$ m.



c) Distance of $\vec{r}_{12} = |\vec{r}_{12}| = \sqrt{1^2+8^2+1^2} = \sqrt{66}$

$= 8.124\ m.$