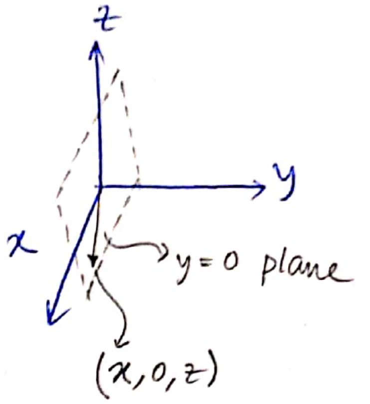
1.) $E = 5xy\,\hat{a_x} + 6(x^2+3)\,\hat{a_y} + 8z^2\,\hat{a_z}$ V/m.

a) Magnitude of $\bar{E}$ in $y=0$ plane.

$\bar{E}(x,0,z) = 6(x^2+3)\hat{a_y} + 8z^2\hat{a_z}$. (Put $y=0$).



$|\bar{E}(x,0,z)|$

$= \sqrt{36(x^2+3)^2 + 64z^4}$ V/m.

b) The value of $\bar{E}$ at $r(1,1,1)$ m.

$\bar{E}(1,1,1) = \cancel{6}\,5.1.1\,\hat{a_x} + 6(1^2+3)\hat{a_y} + 8.1^2\hat{a_z}$

$= 5\hat{a_x} + 24\hat{a_y} + 8\hat{a_z}$

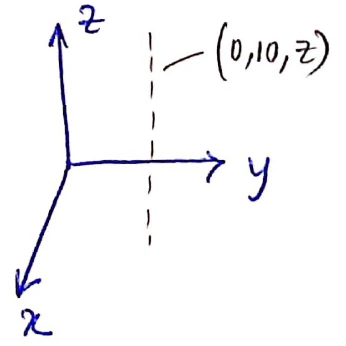
$= (5, 24, 8)$ V/m.

c) The vector component of $\bar{E}$ parallel to y axis

$= \bar{E}.\hat{a_y} = E_y\hat{a_y} = 6(x^2+3)\hat{a_y}$ V/m.

d) $\bar{E}(0,10,z) = 5.0.10\,\hat{a_x} + 6(0^2+3)\hat{a_y} + 8.z^2\hat{a_z}$

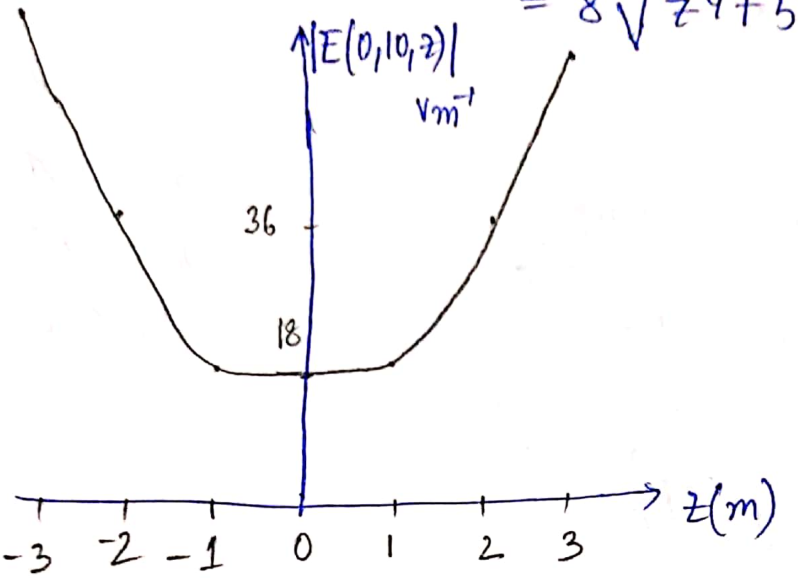
$= 18\hat{a_y} + 8z^2\hat{a_z}$



$$|E(0,10,z)| = \sqrt{18^2 + (8z^2)^2}$$

$$= \sqrt{64z^4 + 324} = \sqrt{64(z^4 + 5.0625)}$$

$$= 8\sqrt{z^4 + 5.0625}\ \text{V/m}$$



2.) Problem 2.7 (b).

$$\bar{G} = (x^2+y^2)\left[\frac{x\,\hat{a_x}}{\sqrt{x^2+y^2+z^2}} + \frac{y\,\hat{a_y}}{\sqrt{x^2+y^2+z^2}} + \frac{z\,\hat{a_z}}{\sqrt{x^2+y^2+z^2}}\right].$$

$$G_\rho = \bar{G}\cdot\hat{a_\rho}$$

$$= (G_x\hat{a_x} + G_y\hat{a_y} + G_z\hat{a_z})\cdot\hat{a_\rho}$$

$$= G_x\cos\phi + G_y\sin\phi$$

$$= \frac{(x^2+y^2)x}{\sqrt{x^2+y^2+z^2}}\cos\phi + \frac{y(x^2+y^2)}{\sqrt{x^2+y^2+z^2}}\sin\phi.$$

$$\boxed{\begin{aligned} x &= \rho\cos\phi \\ y &= \rho\sin\phi \\ x^2+y^2 &= \rho^2 \end{aligned}}$$

$$= \cancel{\frac{\rho^3\cos^2\phi}{\sqrt{\rho^2+z^2}}} + \cancel{\frac{\rho^3\sin^2\phi}{\sqrt{\rho^2+z^2}}}$$

$$G_\phi = G_x(-\sin\phi) + G_y\cos\phi$$

$$= \frac{-x(x^2+y^2)}{\sqrt{x^2+y^2+z^2}}\sin\phi + \frac{y(x^2+y^2)}{\sqrt{x^2+y^2+z^2}}\cos\phi$$

$$G_z = G_z = \frac{(x^2+y^2)z}{\sqrt{x^2+y^2+z^2}}.$$

$x = \rho\cos\phi$
$y = \rho\sin\phi$
$x^2+y^2 = \rho^2.$

$$G_\rho = \frac{(x^2+y^2)\,x\cos\phi}{\sqrt{x^2+y^2+z^2}} + \frac{(x^2+y^2)\,y\sin\phi}{\sqrt{x^2+y^2+z^2}}$$

$$= \frac{x^2+y^2}{\sqrt{x^2+y^2+z^2}}\left\{x\cos\phi + y\sin\phi\right\}$$

$$= \frac{\rho^2}{\sqrt{\rho^2+z^2}}\left\{\rho\cos^2\phi + \rho\sin^2\phi\right\} = \frac{\rho^3}{\sqrt{\rho^2+z^2}}$$

$$G_\phi = \frac{x^2+y^2}{\sqrt{x^2+y^2+z^2}}\left\{-x\sin\phi + y\cos\phi\right\}$$

$$= \frac{\rho^2}{\sqrt{\rho^2+z^2}}\left\{-\rho\cos\phi\sin\phi + \rho\sin\phi\cos\phi\right\} = 0.$$

$$G_z = \frac{\rho^2 z}{\sqrt{\rho^2+z^2}}$$

$$\therefore \bar{G} = \frac{\rho^3}{\sqrt{\rho^2+z^2}}\hat{a}_\rho + \frac{\rho^2 z}{\sqrt{\rho^2+z^2}}\hat{a}_z$$

Cartesian to Spherical Coordinates

$$G_r = \bar{G}\cdot\hat{a}_r = G_x \sin\theta\cos\phi + G_y \sin\theta\sin\phi + G_z\cos\theta$$

$$= \frac{(x^2+y^2)x}{\sqrt{x^2+y^2+z^2}}\sin\theta\cos\phi + \frac{(x^2+y^2)y}{\sqrt{x^2+y^2+z^2}}\sin\theta\sin\phi + \frac{(x^2+y^2)z}{\sqrt{x^2+y^2+z^2}}\cos\theta$$

$$\boxed{\begin{aligned} x &= r\sin\theta\cos\phi \\ y &= r\sin\theta\sin\phi \\ z &= r\cos\theta \end{aligned}}$$

$$= \frac{r^2\sin^2\theta\,\sin\theta\cos\phi}{r}\cdot r\sin\theta\cos\phi + \frac{r^2\sin^2\theta}{r}\cdot r\sin\theta\sin\phi\cdot\sin\theta\sin\phi + \frac{r^2\sin^2\theta}{r}\cdot r\cos\theta\cos\theta$$

$$= r^2\sin^2\theta\,\sin^2\theta\cos^2\phi + r^2\sin^2\theta\sin^2\theta\sin^2\phi + r^2\sin^2\theta\cos^2\theta$$

$$= r^2\sin^2\theta\sin^2\theta + r^2\sin^2\theta\cos^2\theta$$

$$= r^2\sin^2\theta$$

$$G_\theta = G_x \cos\theta \cos\phi + G_y \cos\theta \sin\phi - G_z \sin\theta$$

$$= \frac{(x^2+y^2)x}{\sqrt{x^2+y^2+z^2}} \cos\theta \cos\phi + \frac{(x^2+y^2)y}{\sqrt{x^2+y^2+z^2}} \cos\theta \sin\phi - \frac{(x^2+y^2)z}{\sqrt{x^2+y^2+z^2}} \sin\theta.$$

$$= \frac{r^2\sin^2\theta}{r} . r\sin\theta\cos\phi . \cos\theta . \cos\phi + \frac{r^2\sin^2\theta}{r} . r\sin\theta\sin\phi . \cos\theta\sin\phi - \frac{r^2\sin^2\theta}{r} . r\cos\theta . \sin\theta .$$

$$= r^2\sin^2\theta \sin\theta\cos\theta \cos^2\phi + r^2\sin^2\theta \sin\theta\cos\theta \sin^2\phi - r^2\sin^2\theta \sin\theta\cos\theta .$$

$$= r^2\sin^2\theta \sin\theta\cos\theta - r^2\sin^2\theta\sin\theta\cos\theta$$

$$= 0.$$

$$G_\phi = -G_x \sin\phi + G_y \cos\phi$$

$$= -\frac{x(x^2+y^2)}{\sqrt{x^2+y^2+z^2}} \sin\phi + \frac{(+y)(x^2+y^2)}{\sqrt{x^2+y^2+z^2}} \cos\phi$$

$$= -\frac{r^2\sin^2\theta}{r} . r\sin\theta\cos\phi . \sin\phi + \frac{r^2\sin^2\theta}{r} . r\sin\theta\sin\phi . \cos\phi .$$

$$= 0.$$

$\bar{G} = r^2 \sin^2\theta \, \hat{a}_r$ → Answer.

2.b) Convert $\bar{G} = \rho \sin\phi \, \hat{a}_\rho - \rho \cos\phi \, \hat{a}_\phi + \rho \hat{a}_z$ to rectangular coordinates

In rectangular coordinates,

$$\bar{G} = G_x \hat{a}_x + G_y \hat{a}_y + G_z \hat{a}_z$$

$\hat{a}_\rho \cdot \hat{a}_x = \cos\phi$

$\hat{a}_\phi \cdot \hat{a}_x = -\sin\phi$

$\hat{a}_z \cdot \hat{a}_x = 0$

$G_x = \bar{G} \cdot \hat{a}_x$

$= \rho \sin\phi \, \hat{a}_\rho \cdot \hat{a}_x - \rho \cos\phi \, \hat{a}_\phi \cdot \hat{a}_x + \rho \, \hat{a}_z \cdot \hat{a}_x$

$= \rho \sin\phi \cos\phi + \rho \sin\phi \cos\phi$

$= 2\rho \sin\phi \cos\phi.$

$\hat{a}_\rho \cdot \hat{a}_y = \sin\phi$

$\hat{a}_\phi \cdot \hat{a}_y = \cos\phi$

$\hat{a}_z \cdot \hat{a}_y = 0$

$G_y = \bar{G} \cdot \hat{a}_y$

$= \rho \sin\phi \, \hat{a}_\rho \cdot \hat{a}_y - \rho \cos\phi \, \hat{a}_\phi \cdot \hat{a}_y + \rho \, \hat{a}_z \cdot \hat{a}_y$

$= \rho \sin^2\phi - \rho \cos^2\phi$

$\hat{a}_\phi \cdot \hat{a}_z = 0$

$\hat{a}_\rho \cdot \hat{a}_z = 0$

$G_z = \bar{G} \cdot \hat{a}_z$

$= \rho \sin\phi \, \hat{a}_\rho \cdot \hat{a}_z - \rho \cos\phi \, \hat{a}_\phi \cdot \hat{a}_z + \rho \, \hat{a}_z \cdot \hat{a}_z$

$= \rho$

$\therefore \bar{G} = 2\rho \sin\phi \cos\phi \, \hat{a}_x + \rho(\sin^2\phi - \cos^2\phi) \, \hat{a}_y + \rho \hat{a}_z$

$$\bar{G} = \frac{2\rho\sin\phi\cdot\rho\cos\phi}{\rho}\hat{a_x} + \frac{\rho^2(\sin^2\phi-\cos^2\phi)}{\rho}\hat{a_y} + \rho\hat{a_z}$$

$x^2+y^2=\rho^2$

$x=\rho\cos\phi$

$y=\rho\sin\phi$.

$$= \frac{2xy}{\sqrt{x^2+y^2}}\hat{a_x} + \frac{y^2-x^2}{\sqrt{x^2+y^2}}\hat{a_y} + \sqrt{x^2+y^2}\,\hat{a_z}$$

2c) Convert $\bar{H} = \cos\theta\,\hat{a_r} + \sin\theta\,\hat{a_\theta}$ to rectangular coordinates

$$\bar{H} = H_x\hat{a_x} + H_y\hat{a_y} + H_z\hat{a_z}$$

$\hat{a_r} = \sin\theta\cos\phi\,\hat{a_x} + \sin\theta\sin\phi\,\hat{a_y} + \cos\theta\,\hat{a_z}$

$\hat{a_\theta} = \cos\theta\cos\phi\,\hat{a_x} + \cos\theta\sin\phi\,\hat{a_y} - \sin\theta\,\hat{a_z}$

$$H_x = \bar{H}\cdot\hat{a_x}$$

$$= \cos\theta\,\hat{a_r}\cdot\hat{a_x} + \sin\theta\,\hat{a_\theta}\cdot\hat{a_x}.$$

$$= \cos\theta\cdot\sin\theta\cos\phi + \sin\theta\cos\theta\cos\phi$$

$$= 2\sin\theta\cos\theta\cos\phi.$$

$$\boxed{\begin{aligned}\hat{a_r}\cdot\hat{a_x} &= \sin\theta\cos\phi\\ \hat{a_\theta}\cdot\hat{a_x} &= \cos\theta\cos\phi\end{aligned}}$$

$$H_y = \bar{H}\cdot\hat{a_y}$$

$$= \cos\theta\,\hat{a_r}\cdot\hat{a_y} + \sin\theta\,\hat{a_\theta}\cdot\hat{a_y}$$

$$= \cos\theta\sin\theta\sin\phi + \sin\theta\cos\theta\sin\phi$$

$$= 2\sin\theta\cos\theta\sin\phi$$

$\hat{a_r}\cdot\hat{a_y} = \sin\theta\sin\phi$

$\hat{a_\theta}\cdot\hat{a_y} = \cos\theta\sin\phi$

$H_z = \overline{H}\cdot\hat{a}_z$

$\hat{a}_r\cdot\hat{a}_z = \cos\theta$

$\hat{a}_\theta\cdot\hat{a}_z = -\sin\theta$

$= \cos\theta\,\hat{a}_r\cdot\hat{a}_z + \sin\theta\,\hat{a}_\theta\cdot\hat{a}_z$

$= \cos^2\theta - \sin^2\theta.$

$x = r\sin\theta\cos\phi$

$y = r\sin\theta\sin\phi$

$z = r\cos\theta.$

$x^2+y^2+z^2 = r^2.$

$\sin\theta = \sqrt{\dfrac{x^2+y^2}{r^2}} = \sqrt{\dfrac{x^2+y^2}{x^2+y^2+z^2}}$

$\cos\phi = \dfrac{x}{r\sin\theta}$

$= \dfrac{x}{r\left(\sqrt{x^2+y^2}/r\right)} = \dfrac{x}{\sqrt{x^2+y^2}}.$

$\cos\theta = \dfrac{z}{r} = \dfrac{z}{\sqrt{x^2+y^2+z^2}}$

$\sin\phi = \dfrac{y}{\sqrt{x^2+y^2}}$

$H_x = 2\sin\theta\cos\theta\cos\phi = \cancel{2\sqrt{\dfrac{x^2+y^2}{x^2+y^2+z^2}}}$

$= 2\sqrt{\dfrac{x^2+y^2}{x^2+y^2+z^2}}\cdot\dfrac{z}{\sqrt{x^2+y^2+z^2}}\cdot\dfrac{x}{\sqrt{x^2+y^2}}$

$= \dfrac{2xz}{\sqrt{x^2+y^2+z^2}}$

$H_z = \cos^2\theta - \sin^2\theta$

$= \dfrac{z^2}{x^2+y^2+z^2} - \dfrac{x^2+y^2}{x^2+y^2+z^2}$

$= \dfrac{z^2-x^2-y^2}{x^2+y^2+z^2}$

$$H_y = 2\sin\theta\cos\theta \cdot \sin\phi$$

$$= 2\sqrt{\frac{x^2+y^2}{x^2+y^2+z^2}} \cdot \frac{z}{\sqrt{x^2+y^2+z^2}} \cdot \frac{y}{\sqrt{x^2+y^2}}$$

$$= \frac{2yz}{x^2+y^2+z^2}$$

$$\bar{H} = \frac{1}{x^2+y^2+z^2}\left[2xz\,\hat{a_x} + 2yz\,\hat{a_y} + (z^2-x^2-y^2)\hat{a_z}\right]$$

3.) <u>Prob 2.24(b)</u>

Convert to Cartesian coordinates

$$(3, \pi/2, -1) \rightarrow (x_1, y_1, z_1) \equiv (0, 3, -1) = \vec{r}_1$$

$\rho = 3$

$\phi = \pi/2$

$z = -1.$

$x = \rho\cos\phi = 3\cos\pi/2 = 0$

$y = \rho\sin\phi = 3\sin\pi/2 = 3.$

$z = -1.$

$$(5, 3\pi/2, 5) \rightarrow (x_2, y_2, z_2) \equiv (0, -5, 5) = \vec{r}_2$$

$x_2 = 5\cos(3\pi/2) = 0$

$y_2 = 5\sin(3\pi/2) = -5$

$z_2 = 5$

$\vec{r_{12}} = \vec{r_2} - \vec{r_1}$

$= (0,-5,5) - (0,3,-1)$

$= (0,-8,6)$ m.

$|\vec{r_{12}}| = \sqrt{8^2+6^2} = 10$m. (Ans).

Prob 2.24(c)

$(10, \pi/4, 3\pi/4) \rightarrow (x_1, y_1, z_1)$

$\downarrow r \quad \downarrow \theta \quad \downarrow \phi$.

$$\boxed{\begin{aligned} x &= r\sin\theta\cos\phi \\ y &= r\sin\theta\sin\phi \\ z &= r\cos\theta \end{aligned}}$$

$x_1 = 10\sin(\pi/4)\cos(3\pi/4) = 10 \cdot \frac{1}{\sqrt{2}}\left(-\frac{1}{\sqrt{2}}\right) = -5$

$y_1 = 10\sin(\pi/4)\sin(3\pi/4) = 10 \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = 5$

$z_1 = 10\cos(\pi/4) = \frac{10}{\sqrt{2}} = 5\sqrt{2}$

$(x_1, y_1, z_1) = \vec{r_1} = (-5, 5, 5\sqrt{2})$.

$(5, \pi/6, 7\pi/4) \rightarrow (x_2, y_2, z_2)$

$x_2 = 5\sin(\pi/6)\cos(7\pi/4) = 5 \cdot \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = 1.25\sqrt{2}$

$y_2 = 5\sin(\pi/6)\sin(7\pi/4) = 5 \cdot \frac{1}{2}\left(-\frac{\sqrt{2}}{2}\right) = -1.25\sqrt{2}$

$z_2 = 5\cos(\pi/6) = 5\cdot\frac{\sqrt{3}}{2} = 2.5\sqrt{3}.$

$\vec{r_2} = (1.25\sqrt{2}, -1.25\sqrt{2}, 2.5\sqrt{3})$

$\vec{r_{12}} = \vec{r_2} - \vec{r_1} = (1.25\sqrt{2}, -1.25\sqrt{2}, 2.5\sqrt{3}) - (-5, 5, 5\sqrt{3})$

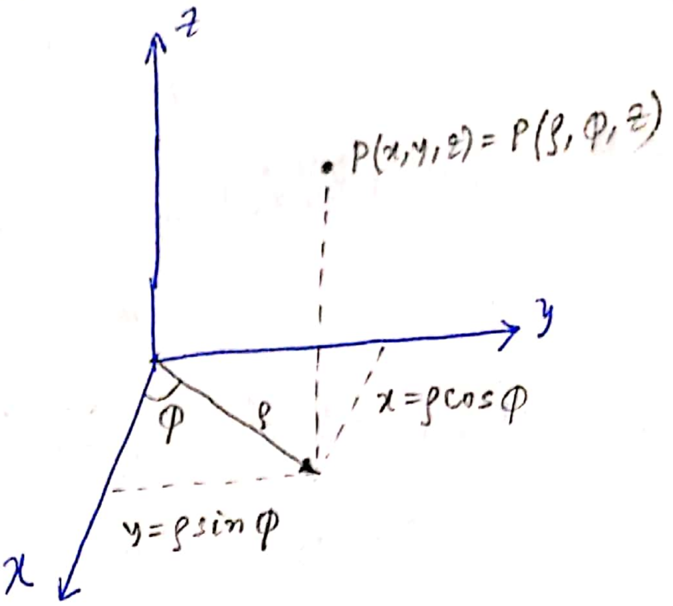
$= (6.7677, -6.7677, -2.7409)$

$|\vec{r_{12}}| = \sqrt{6.7677^2 + 6.7677^2 + 2.7409^2}$

$= 9.955 \text{ m}.$

Appendix Homework 7 EE 331

Conversion of x, y, z Cartesian coordinates to cylindrical (ρ, ϕ, z) coordinates.



Let, $\bar{G} = G_x \hat{a_x} + G_y \hat{a_y} + G_z \hat{a_z}$

be a vector expressed in Cartesian coordinates.

In order to express $\bar{G}$ in cylindrical coordinates →

$$\bar{G} = G_\rho \hat{a_\rho} + G_\phi \hat{a_\phi} + G_z \hat{a_z}$$

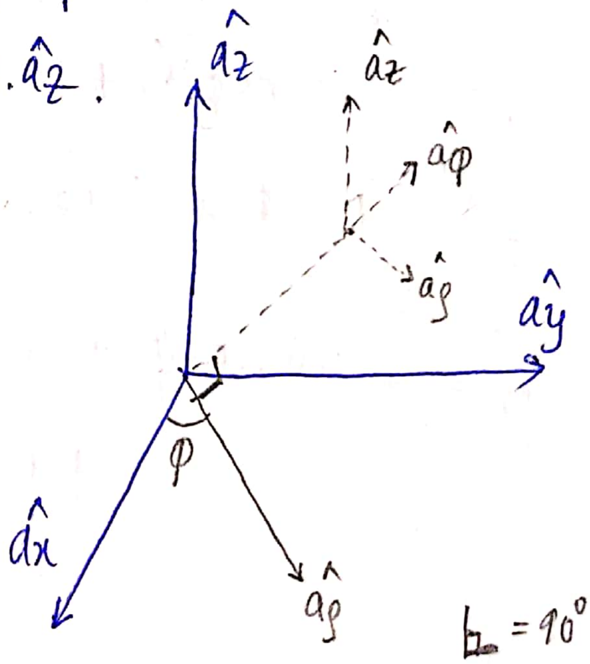
First step

① Take projection of $\bar{G}$ on unit vectors $\hat{a_\rho}$, $\hat{a_\phi}$ and $\hat{a_z}$. which will give G_ρ, G_ϕ, G_z ~~respectively~~. respectively

$$\therefore \quad G_\rho = \bar{G}\cdot\hat{a_\rho} = (G_x\hat{a_x} + G_y\hat{a_y} + G_z\hat{a_z})\cdot\hat{a_\rho}$$

$$G_\phi = \bar{G}\cdot\hat{a_\phi} = (G_x\hat{a_x} + G_y\hat{a_y} + G_z\hat{a_z})\cdot\hat{a_\phi}$$

$$G_z = \bar{G}\cdot\hat{a_z} = (G_x\hat{a_x} + G_y\hat{a_y} + G_z\hat{a_z})\cdot\hat{a_z}.$$



$\hat{a_x}\cdot\hat{a_\rho} = \cos\phi$

$\hat{a_x}\cdot\hat{a_\phi} = \cos(90^\circ + \phi) = -\sin\phi$

$\hat{a_x}\cdot\hat{a_z} = 0.$

$\hat{a_y}\cdot\hat{a_\rho} = \cos(90^\circ - \phi) = \sin\phi$

$\hat{a_y}\cdot\hat{a_\phi} = \cos\phi$ $\hat{a_y}\cdot\hat{a_z} = 0.$

$$G_\rho = (G_x \hat{a}_x + G_y \hat{a}_y + G_z \hat{a}_z) \cdot \hat{a}_\rho$$
$$= G_x \cos\phi + G_y \sin\phi$$

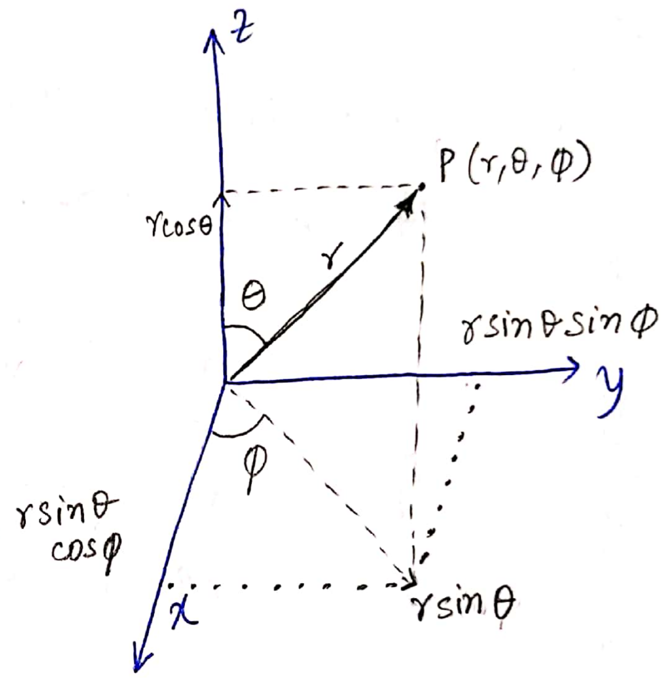
$$\hat{a}_z \cdot \hat{a}_\rho = \cos 90^\circ = 0$$
$$\hat{a}_z \cdot \hat{a}_\phi = 0$$
$$\hat{a}_z \cdot \hat{a}_z = 1.$$

$$G_\phi = (G_x \hat{a}_x + G_y \hat{a}_y + G_z \hat{a}_z) \cdot \hat{a}_\phi$$
$$= G_x (-\sin\phi) + G_y \cos\phi$$

$$G_z = (G_x \hat{a}_x + G_y \hat{a}_y + G_z \hat{a}_z) \cdot \hat{a}_z$$
$$= 0 + 0 + G_z. = G_z.$$

Conversion from Cartesian (x, y, z) to spherical coordinates (r, θ, ϕ)

$$\bar{G} = G_x \hat{a}_x + G_y \hat{a}_y + G_z \hat{a}_z$$
$$\bar{G} = G_r \hat{a}_r + G_\theta \hat{a}_\theta + G_\phi \hat{a}_\phi$$



$$G_r = \bar{G} \cdot \hat{a}_r$$
$$= (G_x \hat{a}_x + G_y \hat{a}_y + G_z \hat{a}_z) \cdot \hat{a}_r.$$

$$\hat{a}_r = \sin\theta\cos\phi \, \hat{a}_x + \sin\theta\sin\phi \, \hat{a}_y + \cos\theta \, \hat{a}_z.$$

$$\cancel{\hat{a}_x = \hat{a}_x \sin\theta\cos\phi}$$
$$\cancel{\hat{a}_y = \hat{a}_r}$$

$$G_r = G_x \sin\theta\cos\phi + G_y \sin\theta\sin\phi + G_z \cos\theta.$$

$= \cancel{(G_x a_x + G_y a_y + G_z a_z) \cdot a_\theta}$

From above diagram,

$x = r\sin\theta\cos\phi$ $\quad y = r\sin\theta\sin\phi$ $\quad z = r\cos\theta$

$\frac{dx}{r\,d\theta} = \cos\theta\cos\phi$ $\quad \frac{dy}{r\,d\theta} = \cos\theta\sin\phi$ $\quad \frac{dz}{r\,d\theta} = -\sin\theta$

$\therefore \hat{a_\theta} = \cos\theta\cos\phi\,\hat{a_x} + \cos\theta\sin\phi\,\hat{a_y} - \sin\theta\,\hat{a_z}.$

$G_\theta = \bar{G}\cdot\hat{a_\theta}$

$= G_x\cos\theta\cos\phi + G_y\cos\theta\sin\phi - G_z\sin\theta$

$G_\phi = \bar{G}\cdot\hat{a_\phi}$

$\frac{dx}{r\sin\theta\,d\phi} = -\sin\phi$ $\quad \frac{dy}{r\sin\theta\,d\phi} = \cos\phi$ $\quad \frac{dz}{r\sin\theta\,d\phi} = 0.$

$\therefore \hat{a_\phi} = -\sin\phi\,\hat{a_x} + \cos\phi\,\hat{a_y}.$

$G_\phi = -G_x\sin\phi + G_y\cos\phi.$