EE 331 - LECT #10 FEB. 7, 2020

we can now find $V_s(z)$ & $I_s(z)$ anywhere along a lossy TL using:

$$V_s(z) = V_0^+\left(e^{-\gamma z} + \Gamma_L e^{\gamma z}\right) \; [V]$$

$$I_s(z) = \frac{V_0^+}{Z_0}\left(e^{-\gamma z} - \Gamma_L e^{\gamma z}\right) \; [A]$$

with

$$V_0^+ = \left(\frac{Z_{in}}{Z_{in}+Z_g}\right)\frac{V_g}{e^{\gamma \ell} + \Gamma_L e^{-\gamma \ell}} = |V_0^+| e^{j\phi^+} \; [V]$$

$$Z_{in} = Z_0\left[\frac{Z_L + Z_0 \tanh(\gamma \ell)}{Z_0 + Z_L \tanh(\gamma \ell)}\right] \; [\Omega]$$

$$Z_0 = \sqrt{\frac{(R+j\omega L)}{(G+j\omega C)}} \; [\Omega]$$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} \; [1/m] = \alpha + j\beta$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = |\Gamma_L| e^{j\theta_r} \; [\cdot]$$

for typical freqs used w/ TL's (high freqs require waveguides), we can ignore loss because it's relatively small. at high freqs, TL's are too lossy.

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lossless lines - $R=0, G=0$

what happens to Z_{in}, Z_0, & γ?

$$\gamma = \sqrt{(0+j\omega L)(0+j\omega C)} = \sqrt{-\omega^2 LC} = j\omega\sqrt{LC}$$
$$= \alpha + j\beta$$

$$\rightarrow \alpha = 0, \ \beta = \omega\sqrt{LC}$$

this makes sense because shouldn't have attenuation in lossless line ($\alpha = 0$)

$$Z_0 = \sqrt{\frac{0+j\omega L}{0+j\omega C}} = \sqrt{L/C} \ [\Omega] \rightarrow \text{real}$$

$$Z_{in} = Z_0\left(\frac{Z_L - jZ_0\tan(\beta z)}{Z_0 - jZ_L\tan(\beta z)}\right) \quad \tanh(jx) = j\tan(x)$$

also,

$$V_0^+ = \left(\frac{Z_{in}}{Z_g + Z_{in}}\right)\frac{V_g}{(e^{j\beta l} + \Gamma_L e^{-j\beta l})} = |V_0^+|e^{j\phi} \ [V]$$

$$V_s(z) = V_0^+(e^{-j\beta z} + \Gamma_L e^{j\beta z}) \ [V]$$

$$I_s(z) = \frac{V_0^+}{Z_0}(e^{-j\beta z} - \Gamma_L e^{j\beta z}) \ [A]$$

V_0^+, Γ_L, Z_{in}, V_g, Z_L can still be complex.

standing waves - lossless line

traveling incident & reflected waves combine to form a standing wave. what is magn of $V_s(z)$ along the line?

recall, $z = x + jy = |z| e^{j\theta}$

$z^* = x - jy = |z| e^{-j\theta}$

$\longrightarrow zz^* = |z|^2 \longrightarrow |z| = \sqrt{zz^*}$

thus,

$$|V_s(z)| = [V_s(z) V_s^*(z)]^{1/2}$$

$$= \{|V_0^+| e^{j\phi}(e^{-j\beta z} + |\Gamma_L| e^{j\theta_r + j\beta z}) |V_0^+| e^{-j\phi}(e^{j\beta z} + |\Gamma_L| e^{-j\theta_r - j\beta z})\}^{1/2}$$

(bunch of algebra)

$$= |V_0^+| \{1 + |\Gamma_L|^2 + 2|\Gamma_L| \cos(\underbrace{\theta_r + 2\beta z}_{\text{varies periodically along line}})\}^{1/2}$$

const ≥ 0

min & max every $\lambda/2$ because of the $2\beta z$:

$$2\beta z = 2 \cdot \frac{2\pi}{\lambda}\left(\frac{n\lambda}{2}\right) = 2n\pi$$

max magn when $\cos(\,) = 1$; min when $\cos(\,) = -1$

$$|V_s|_{max} = |V_0^+|[1 + |\Gamma_L|^2 + 2|\Gamma_L|]^{1/2}$$

$$= |V_0^+|(1 + |\Gamma_L|) = \text{max value of magn}$$

$$|V_s|_{min} = |V_0^+|(1 - |\Gamma_L|) = \text{min voltage magn}$$

standing wave ratio

define:

$$s = \frac{|V_s|_{max}}{|V_s|_{min}} = \boxed{\frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} = s}$$

= voltage standing wave ratio

= SWR or VSWR

note that:

$$\boxed{|\Gamma_L| = \frac{s - 1}{s + 1}}$$