EE 331 - LECT #15 FEB. 21, 2020

vectors & vector algebra

a vector $\bar{A}$ has a magnitude $|\bar{A}|$, A & direction $\hat{a}_A$. given:

$$\bar{A} = 2\hat{a}_x + 3\hat{a}_y + 4\hat{a}_z \qquad (2,3,4)$$
$$\bar{B} = \hat{a}_x + 2\hat{a}_y + 3\hat{a}_z \qquad (1,2,3)$$

a) $\bar{A} - \bar{B} = \hat{a}_x + \hat{a}_y + \hat{a}_z = (1,1,1)$

b) $|\bar{A} + \bar{B}| = \sqrt{83} \approx 9.11$

note:

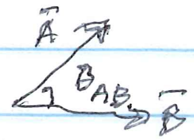
1. if $\bar{C} = \bar{A} + \bar{B}$, then
$$C_x = A_x + B_x$$
$$C_y = A_y + B_y$$
$$C_z = A_z + B_z$$

→ 1 vector eq = 3 scalar eqs; true for any vector or vector quantity including diff eqs

2. $\bar{A} \cdot \bar{B}$ always a scalar
3. $\bar{A} \times \bar{B}$ " " vector
4. unit vector in direction of vector, e.g.,
$$\hat{a}_A = \frac{\bar{A}}{A} \quad \text{where } A = |\bar{A}|$$

vector multiplication

dot or scalar product - always gives scalar:



$$\bar{A} \cdot \bar{B} = A_x B_x + A_y B_y + A_z B_z$$
$$= AB \cos\theta_{AB}$$

$$\cos\theta_{AB} = \frac{\bar{A}\cdot\bar{B}}{AB} \rightsquigarrow \theta_{AB} = \cos^{-1}\left(\frac{\bar{A}\cdot\bar{B}}{AB}\right)$$

given $\bar{A} = (2, 3, 4)$ & $\bar{B} = (1, 2, 3)$ find θ_{AB}

$$\theta_{AB} = 6.9825°$$

cross or vector product = gives vector!

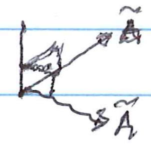
$$\bar{A}\times\bar{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_yB_z - A_zB_y)\,\hat{a}_x + (A_zB_x - A_xB_z)\,\hat{a}_y + (A_xB_y - B_yB_x)\,\hat{a}_z$$

$\hat{a}_z \to \hat{a}_y \to \hat{a}_x \to \hat{a}_z$

$= AB\sin\theta_{AB}\,\hat{a}_n$

= vector $\perp$ to plane formed by $\bar{A}$ & $\bar{B}$ using RHR

if $\bar{A}\times\bar{B} = \bar{C}$, right angles btwn $\bar{A}$ & $\bar{C}$ and $\bar{B}$ & $\bar{C}$



vector division

$$\frac{\bar{A}}{\bar{B}} = ?$$ nonsense

you can't divide a vector by another vector !!!