EE 331 – LECT. #17 FEB. 26, 2020

PSS – MON. 5:10 – 6 PM, SLOAN 161

NEW TA OH: M 11–12, TH 1–2

vector fields

defn: a vector field is a vector func that changes both in magnitude & direction as a func of position &/or time

e.g.,

$$\bar{G}(\bar{r}) = G_x(\bar{r})\hat{a}_x + G_y(\bar{r})\hat{a}_y + G_z(\bar{r})\hat{a}_z$$

where

$G_x\hat{a}_x$ = vector component of $\bar{G}$ in x dir
$G_y\hat{a}_y$ = " " " " " y "
$G_z\hat{a}_z$ = " " " " " z "
} all funcs of position

but recall $\bar{r}$ = position vector = (x, y, z) m. so e.g.,

$$\bar{G}(x,y,z) = G_x(x,y,z)\hat{a}_x + G_y(x,y,z)\hat{a}_y + G_z(x,y,z)\hat{a}_z$$

(all funcs of x, y, & z)

e.g.,

$$G_x = y\sin x$$
$$G_y = z\cos x$$
$$G_z = x^2y^2z^2$$

given $\bar{E} = 24xy\,\hat{a}_x + 12x^2\hat{a}_y + 18z^2\hat{a}_z$ V/m, find:

(a) $|E|$ in $z=0$ plane
(b) $\bar{E}$ along line $(0,4,z)$
(c) vector component of $\bar{E}$ in x dir (i.e., parallel to $\hat{a}_x$)
(d) plot $|E|$ along line $(0,4,z)$, $-2 \le z \le 2$

treat vector fields just like other funcs.

e.g., $$\frac{\partial \vec{G}}{\partial x} = \frac{\partial G_x}{\partial x}\hat{a}_x + \frac{\partial G_y}{\partial x}\hat{a}_y + \frac{\partial G_z}{\partial x}\hat{a}_z$$

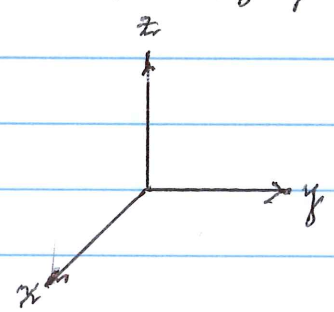
$$\int \vec{G}\,dx = \int G_x \hat{a}_x\,dx + \int G_y \hat{a}_y\,dx + \int G_z \hat{a}_z\,dx$$

ch 2 - coord systems

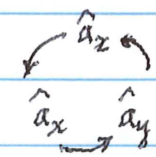
we'll use 3 orthogonal coord systems, i.e., unit vectors are all perpendicular to each other

- cartesian (rectangular)
- cylindrical
- spherical

1. cartesian $P(x, y, z)$

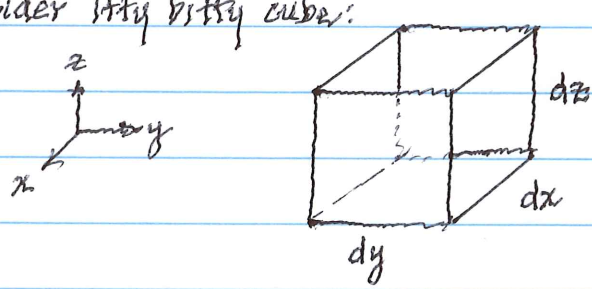


$-\infty < x, y, z < \infty$



- use RHR to set up axes
- $\hat{a}_x, \hat{a}_y, \hat{a}_z$ = const unit vectors
  → magn = 1, dirs don't changes
- position vector: $\vec{r} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$ [m]

consider itty bitty cube:



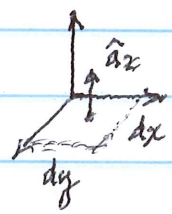
<u>differential quantities</u>

rect: 1) differential length $\overline{dl}$ [m] = vector so treat like vector

$$\overline{dl} = dx\hat{a}_x + dy\hat{a}_y + dz\hat{a}_z \ [m]$$

2) differential surf area $\overline{ds}$ = vector so treat like vector

$$\overline{ds} = dxdy\,\hat{a}_z \ [m^2]$$
$$\overline{ds} = dydz\,\hat{a}_x \ [m^2]$$
$$\overline{ds} = dxdz\,\hat{a}_y \ [m^2]$$



$ds = dxdy \ [m^2]$

- positive dir points outward from vol

3) differential vol dv

$$dv = dx\,dy\,dz \ [m^3]$$

1. $|\overline{dl}| =$
2. $\overline{A}\cdot\overline{dl} =$
3. $\overline{A}\cdot\overline{ds}$ w/ $\overline{ds}$ parallel to the $+x$ axis