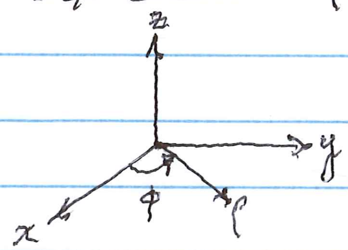
EE 331 - LECT. #18 FEB. 28, 2020

coord systems (cont.)

2. cylindrical $P(\rho, \phi, z)$



$0 \le \rho < \infty$

$0 \le \phi < 2\pi$, measured from $+x$ axis

$-\infty < z < \infty$

- use RHR to set up axes
- $\hat{a}_\rho$ & $\hat{a}_\phi \ne$ const unit vectors
  - $\rightarrow$ magn = 1, but dirs change
- position vector: $\bar{r} = \rho \hat{a}_\rho + z \hat{a}_z$ [m]
  - no ϕ !!! ϕ is an angle
  - $\bar{r} \ne (\rho, \phi, z)$ but $\bar{A} = (A_\rho, A_\phi, A_z)$

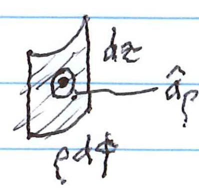
differential quantities

cyl: $\bar{dl} = d\rho\,\hat{a}_\rho + \underbrace{\rho\, d\phi}_{}\,\hat{a}_\phi + dz\,\hat{a}_z$ [m]

= arc length of circle

- recall  $s = r\theta$

$\bar{ds} = \pm \rho\, d\phi\, dz\, \hat{a}_\rho$ [m^2]



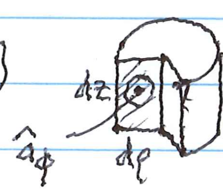
$\odot$ out of surf

$\otimes$ into surf

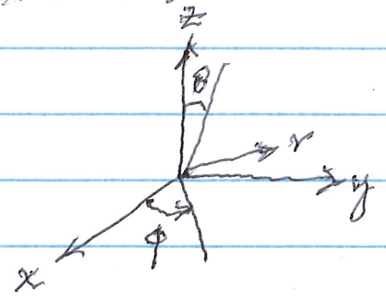
+ out of vol

− into vol

$\bar{ds} = \pm d\rho\, dz\, \hat{a}_\phi$ [m^2]



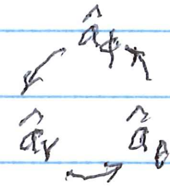
3. spherical coords $P(r, \theta, \phi)$



$0 \leq r < \infty$

$0 \leq \theta \leq \pi$

$0 \leq \phi < 2\pi$



- use RHR to set up axes
- $\hat{a}_r, \hat{a}_\theta, \hat{a}_\phi \neq$ const unit vectors
  - $\rightarrow$ magn $= 1$, but dirs change
- position vector: $\vec{r} = r\hat{a}_r$ [m] no θ or ϕ !!!
  - $\vec{r} \neq (r, \theta, \phi)$ but $\vec{A} = (A_r, A_\theta, A_\phi)$

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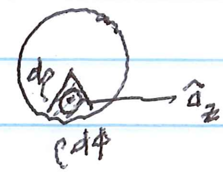
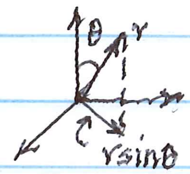
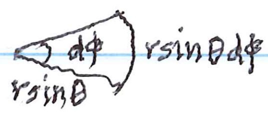
$\bar{ds} = \pm \rho\, d\rho\, d\phi\, \hat{a}_z$ [m^2]



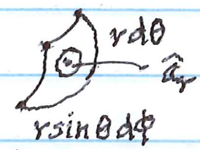
$dv = \rho\, d\rho\, d\phi\, dz$ [m^3]

spher: $\bar{dl} = dr\hat{a}_r + \underbrace{r\,d\theta}_{=\text{ arc length}}\hat{a}_\theta + \underbrace{r\sin\theta}_{\text{radius of circle projection}}d\phi\,\hat{a}_\phi$

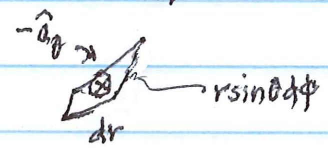




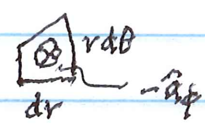
$\bar{ds} = \pm r^2 \sin\theta\, d\theta\, d\phi\, \hat{a}_r$ [m^2]



$\bar{ds} = \pm r\sin\theta\, dr\, d\phi\, \hat{a}_\theta$ [m^2]



$\bar{ds} = \pm r\, dr\, d\theta\, \hat{a}_\phi$ [m^2]



$dv = r^2 \sin\theta\, dr\, d\theta\, d\phi$ [m^3]

coord transforms of vector fields

cart $\leftrightarrow$ cyl

cart $\leftrightarrow$ spher

two steps: 1. transform components of $\bar{A}$, e.g., A_x, A_y, A_z

2. " variables e.g., x, y, z