EE 331 - LECT. #19 MAR. 2, 2020

coord transforms of vector fields - projection method

cart $\leftrightarrow$ cyl

cart $\leftrightarrow$ spher

two steps: 1. transform components, e.g., $(A_x, A_y, A_z) \rightarrow (A_\rho, A_\phi, A_z)$

2. " variables, e.g., $x \rightarrow \rho\cos\phi$

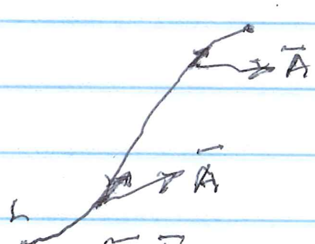
ex. #13

ch. 3 - vector calculus - integration & differentiation

1. line integral (single integral)

defn: the line integral of a vector field $\bar{A}$ along a contour L is given by:

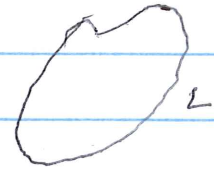
$$\int_L \bar{A} \cdot \bar{d\ell} = \text{single integral} = \text{scalar}$$



$\bar{A} \cdot \bar{d\ell}$ = component of $\bar{A}$ parallel (tangential) to $\bar{d\ell}$

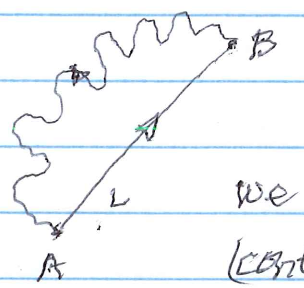
$\int_L \bar{A} \cdot \bar{dl}$ = sum of itty bitty amounts of $\bar{A}$ tangent to contour L

for a closed contour:



$\oint \bar{A} \cdot \bar{dl}$ = $\boxed{\text{circulation}}$ of $\bar{A}$ around L

if $\oint \bar{A} \cdot \bar{dl} = 0$, then $\bar{A}$ is conservative field & integral is path independent



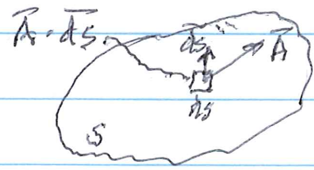
we can change path of integration (contour L) to make problems easier → get same answer

2. surface integral = flux (double integral)

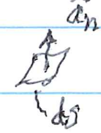
defn: the surface integral or flux of $\bar{A}$ through surface S is given by:

$\int_S \bar{A} \cdot \bar{ds} = \iint$ = double integral = scalar

$\bar{ds}$ indicates it's a double integral



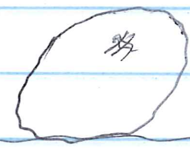
$\bar{A} \cdot \bar{ds}$ = component of $\bar{A}$ perpendicular to surf S



$\bar{ds} = ds\,\hat{a}_n$

$\int_S \bar{A} \cdot \bar{ds}$ = sum of itty bitty amounts of $\bar{A}$ perpendicular to the surf

for a closed surf:



$\oint_S \bar{A} \cdot \bar{ds} = \boxed{\text{net outward flux}}$ of $\bar{A}$ through S

ex. #14