EE-331 - LECT. #20   MAR. 4, 2020

ex. #15

3. volume integral (triple integral)

defn: the volume integral of a scalar quantity is given by:

$$\int_v \rho_v \, dv = \text{triple integral}$$
$$= \text{scalar}$$

for example, given $\rho_v = \frac{1}{\pi r}$ C/m^3, find the total charge enclosed by a sphere of radius 2 m.

$$Q = \int \rho_v \, dv = \int \frac{1}{\pi r} r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$= \frac{1}{\pi} \int_0^2 r \, dr \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\phi$$

$$= \frac{1}{\pi} \frac{r^2}{2} \Big|_0^2 \cos\theta \Big|_{\pi}^{0} \phi \Big|_0^{2\pi}$$

$$= \frac{4}{2\pi} (2)(2\pi) = \boxed{8 \text{ C}}$$

vector differentiation

1. del operator ∇ - derivative equivalent of $\int$

∇ = "del" or "nabla" $\boxed{[1/m]}$!!!

eg

$$\nabla = \frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z \quad [1/m]$$

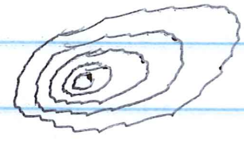
∇ = shorthand for taking the derivative wrt position

eg.

$$\left.\begin{aligned} \nabla \cdot \bar{D} &= \rho_v \\ \nabla \cdot \bar{B} &= 0 \\ \nabla \times \bar{E} &= -\frac{\partial \bar{B}}{\partial t} \\ \nabla \times \bar{H} &= \bar{J} + \frac{\partial \bar{D}}{\partial t} \end{aligned}\right\} \text{maxwell's eqs}$$

2. gradient of a scalar field $\rightarrow$ vector field

$$\text{grad } V = \nabla V$$



- ∇V = vector field
- $|\nabla V|$ = max rate of change in V
- ∇V points in the dir of max rate of change