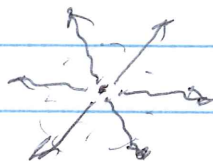


3. divergence of a vector field = scalar field

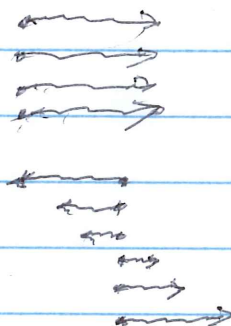
$$\text{div } \vec{A} = \nabla \cdot \vec{A}$$

$$= \lim_{\Delta V \rightarrow 0} \oint \vec{A} \cdot d\vec{s}$$



4. curl of a vector field = vector field

$$\text{curl } \vec{A} = \nabla \times \vec{A}$$



4. Laplacian of a scalar or vector field = scalar or vector field

$$\text{Laplacian of } V = \nabla^2 V$$

$$\text{or } \vec{A} = \nabla^2 \vec{A}$$

two important & very useful vector identities

always true

1) $\nabla \times \nabla V = 0$ curl of grad is zero

2) $\nabla \cdot (\nabla \times \vec{A}) = 0$ div of curl is zero

Two important theorems for vector fields

div thm: $\oint_S \vec{A} \cdot d\vec{s} = \int_V (\nabla \cdot \vec{A}) dV$



Stokes's thm: $\oint_L \vec{A} \cdot d\vec{r} = \int_S (\nabla \times \vec{A}) \cdot d\vec{s}$



div thm: 1) net outward flux of $\vec{A}$ from $S = \text{vol}$
integral of div of $\vec{A}$

2) closed double integral = triple integral

Stokes's thm: 1) circulation of $\vec{A}$ around $L = \text{open surf}$
integral of div of $\vec{A}$

2) closed line integral = open surf integral

ex. A 16