EE 331 - LECT. #23 MAR. 23, 2020

FAQ @ vcea.wsu.edu/student-success/

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EM - FUNDAMENTAL PROBLEM

Q: if i hold up charge here at $\bar{r}'$, what effect does it have over there at $\bar{r}$?

A: it depends on what i do with it

electrostatics - hold charge still → const elec field

thus, given a static charge and/or charge distribution at $\bar{r}'$, goal is to find elec field $\bar{E}$ at $\bar{r}$.

three ways to find $\bar{E}$

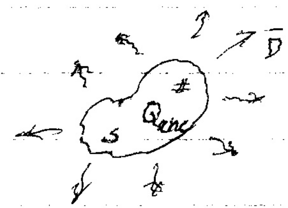
1. C's law (& superposition) } (2 fundamental laws of electrostatics)
2. Gauss's law
3. negative of the gradient of the potential (but then must first find potential)

last lecture, used C's law & superposition to find $\bar{E}$

example #18 redux

gauss's law – derived from coulomb's law

fundamental law of nature



$$\oint_S \bar{D} \cdot \bar{ds} = Q_{enc}$$

net elec flux through a closed surf = charge enclosed by surf

where

$$\bar{D} = \varepsilon \bar{E} \quad [C/m^2]$$

$$= \text{elec flux density}$$

next, use the div thm:

$$\oint_S \bar{D} \cdot \bar{ds} \underset{\text{by div thm}}{=} \int_v (\nabla \cdot \bar{D})\, dv \overbrace{= Q_{enc} =}^{*} \int \rho_v \, dv$$

* true for any vol v – only way this is true is if integrands are equal. thus,

$$\boxed{\nabla \cdot \bar{D} = \rho_v}$$ maxwell's first eq

= gauss's law in diff form

we can use g's law to find $\bar{E}$ but only for very symmetric charge densities. we won't use it.

work

if i have a charge here (which gives rise to an elec field), what must i do to move another charge near it? work!

recall,

$$W = \int \vec{F} \cdot \vec{dl} \quad [J]$$

by defn

$$\vec{E} = \vec{F}/Q \rightarrow \vec{F} = Q\vec{E} = \text{force due to elec field}$$

must move Q against field

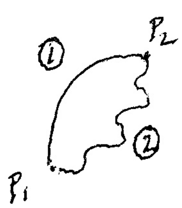
so

$$W = -Q \int_{P_1}^{P_2} \vec{E} \cdot \vec{dl}$$

due to charge Q_1

negative sign because work done against field, i.e., opposite to the direction of the field.

- $W > 0$: work required (rock up hill)
- $W < 0$: " supplied (rock down hill)
- $\vec{E}$ is conservative so line integral is path indep, & net work around closed path is zero.



$$W = \int_{①} = \int_{②} \rightarrow \int_{①} - \int_{②} = 0$$

$$\therefore \; W = -Q \oint_L \vec{E} \cdot \vec{dl} = 0$$

$$\rightarrow \oint_L \vec{E} \cdot \vec{dl} \overset{?}{=} \int_S (\nabla \times \vec{E}) \cdot \vec{ds} = 0$$

thus,

$$\boxed{\nabla \times \vec{E} = 0}$$ maxwell's 2nd eq for electrostatics