EE 331 - LECT. #24 MAR. 25, 2020

recall from monday,

$$W = -Q\int_{P_1}^{P_2} \bar{E}\cdot\bar{dl}$$ = work done moving charge Q from P_1 to P_2

potential

defn: $V = W/Q$ (similar to $\bar{E} = \bar{F}/Q$)

$$\rightarrow V = \frac{-Q\int \bar{E}\cdot\bar{dl}}{Q} = -\int_{P_1}^{P_2} \bar{E}\cdot\bar{dl} = \underbrace{V_2}_{\text{pot @ } P_2} - \overbrace{V_1}^{\text{pot @ } P_1}$$

i.e.,

$$V_{12} = -\int_{P_1}^{P_2} \bar{E}\cdot\bar{dl}$$ = potential difference [V]

V not absolute! it's relative. we usually choose P_1 at ∞ & let V = 0 at ∞, but it depends on the geometry. we can find V for any given charge distribution:

pt charge: $V(\bar{r}) = \frac{1}{4\pi\varepsilon}\frac{Q}{|\bar{r}-\bar{r}'|}$ [V]

line charge: $V(\bar{r}) = \frac{1}{4\pi\varepsilon}\int_L \frac{\rho_L\, dl'}{|\bar{r}-\bar{r}'|}$ [V]

surf charge: $V(\bar{r}) = \frac{1}{4\pi\varepsilon}\int_S \frac{\rho_S\, ds'}{|\bar{r}-\bar{r}'|}$ [V]

vol charge: $V(\bar{r}) = \frac{1}{4\pi\varepsilon}\int_V \frac{\rho_v\, dv'}{|\bar{r}-\bar{r}'|}$ [V]

example #19

next, let's return to the equation for V:

$$V = -\int \bar{E} \cdot \bar{dl}$$

using cartesian coords we take the differential of both sides of this eq:

$$dV = -\bar{E} \cdot \bar{dl} = -(E_x, E_y, E_z) \cdot (dx, dy, dz)$$

$$\text{RHS: } -E_x dx - E_y dy - E_z dz$$

$$\text{LHS: } \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz$$

where,

$\frac{\partial V}{\partial x}$ = rate of change of V in the x dir

dx = tiny distance in the x dir

thus,

$\frac{\partial V}{\partial x} dx$ = change of V in x dir over distance dx

the same is true in the y & z dirs. equating LHS & RHS:

$$-\frac{\partial V}{\partial x} = E_x, \quad -\frac{\partial V}{\partial y} = E_y, \quad -\frac{\partial V}{\partial z} = E_z$$

in vector form:

$$E_x \hat{a}_x + E_y \hat{a}_y + E_z \hat{a}_z = -\frac{\partial V}{\partial x} \hat{a}_x - \frac{\partial V}{\partial y} \hat{a}_y - \frac{\partial V}{\partial z} \hat{a}_z$$

or,

$$\boxed{\bar{E} = -\nabla V \quad \{V/m\}}$$ elec field = −grad of pot

so if we know V, we can find $\bar{E}$.

given charge @ origin, $V = \frac{Q}{4\pi\varepsilon} \frac{1}{r}$ {V}. find $\bar{E}$.