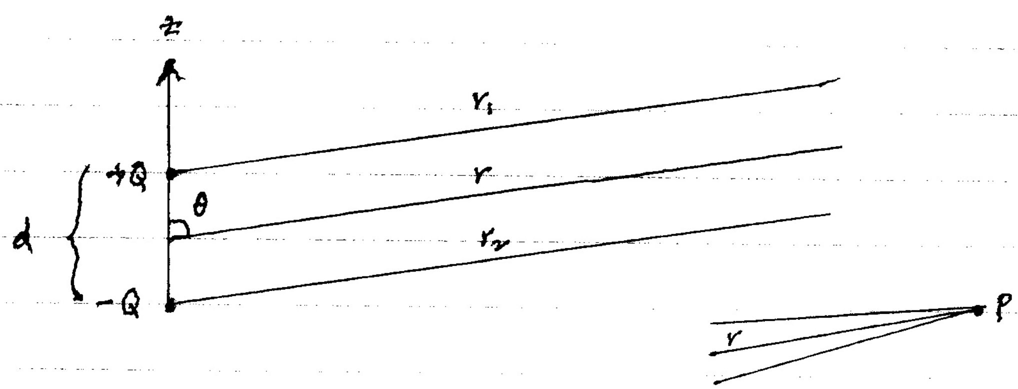
EE 331 – LECT. #25 MAR. 27, 2020

3 ways of finding $\bar{E}$? answer in chat window.

1. coulomb's laws
2. gauss's laws
3. $\bar{E} = -\nabla V$

<u>electric dipole</u>

{defn:} dipole = 2 charges of equal magnitude but of opposite sign separated by a distance d



potentials due to pt charges:

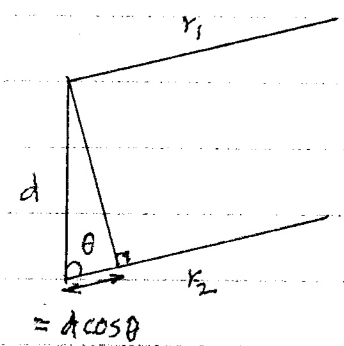
$$V^{+} = \frac{Q}{4\pi\varepsilon}\frac{1}{r_1} \quad [V]$$

$$V^{-} = \frac{-Q}{4\pi\varepsilon}\frac{1}{r_2} \quad [V]$$

$$\rightarrow \quad V = V^{+} + V^{-} = \frac{Q}{4\pi\varepsilon}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$= \frac{Q}{4\pi\varepsilon}\left(\frac{r_2 - r_1}{r_1 r_2}\right) \quad [V]$$

for $r \gg d$:



$\therefore\ r_2 - r_1 \cong d\cos\theta \quad \rightarrow d\cos\theta < d \ll r$

$r_1 r_2 \cong r^2$

pot becomes:

$$V \cong \frac{Q}{4\pi\varepsilon}\,\frac{d\cos\theta}{r^2} \qquad [V]$$

* find $\bar{E}$ for dipole: $-\frac{\partial V}{\partial r}\hat{a}_r - \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{a}_\theta$

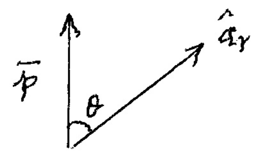
interestingly,

	V	E
pt charge (monopole)	$1/r$	$1/r^2$
dipole (2 mono's)	$1/r^2$	$1/r^3$
quadrupole (2 di's)	$1/r^3$	$1/r^4$
octupole (2 quads)	$1/r^4$	$1/r^5$

defn: dipole moment $= \bar{p} = Q\vec{d}$ [C·m]

magnitude of $\bar{p}$: $p = Qd$

→ $Qd\cos\theta = \bar{p}\cdot\hat{a}_r$ because θ is angle btwn $\bar{p}$ & $\hat{a}_r$



using $\bar{p}$:

$$V = \frac{\bar{p}\cdot\hat{a}_r}{4\pi\varepsilon r^2} \quad [V]$$

$$\bar{E} = \frac{p}{4\pi\varepsilon r^3}(2\cos\theta\,\hat{a}_r + \sin\theta\,\hat{a}_\theta) \quad [V/m]$$

$\bar{E}$ always points from higher to lower potential

$\bar{E}$ & V lines always perpendicular to each other

for dipoles $\bar{E}$ looks like a toroid

dipole for prostate cancer - $E \propto \frac{1}{r^3}$