EE 331 - LECT. #27 APR. 1, 2020

joule's law - power supplied by elec field

$$dW = d\bar{F}\cdot d\bar{l}$$

$= -(dQ\,\bar{E}\cdot d\bar{l})$ $\quad \bar{F} = Q\bar{E}$, neg sign because work against $\bar{E}$

$= -(\rho_v\,dv\,\bar{E}\cdot d\bar{l})$ $\quad dQ = \rho_v\,dv$

$= -(\rho_v\,dv\,\bar{E}\cdot \bar{u}\,dt)$ $\quad d\bar{l} = \bar{u}\,dt$

$= \rho_v\,dv\,\bar{E}\cdot \bar{u}_e\,dt$ $\quad \bar{u}_e = -\bar{u}$

$= \rho_v\,\bar{u}_e\cdot\bar{E}\,dv\,dt$

$= \bar{J}\cdot\bar{E}\,dv\,dt$ $\quad \bar{J} = \rho_v\,\bar{u}_e$

and

$$\frac{dW}{dt} = \bar{J}\cdot\bar{E}\,dv$$

integrate both sides:

$$\int\frac{dW}{dt} = \boxed{P = \int_v \bar{J}\cdot\bar{E}\,dv} \quad \text{joule's law}$$

for straight cond w/ const radius, rewrite power as:

$$P = \int_v \bar{J}\cdot d\bar{s}\int \bar{E}\cdot d\bar{l} \qquad dv = ds\,dl$$

$$= \quad I \qquad V$$

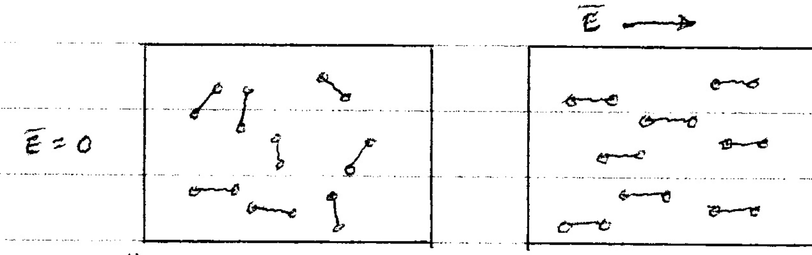
$\therefore P = VI = V^2/R = I^2R$ [W] = joule's law in ckt theory

= special case of joule's law

dielectrics - use simple dipole model

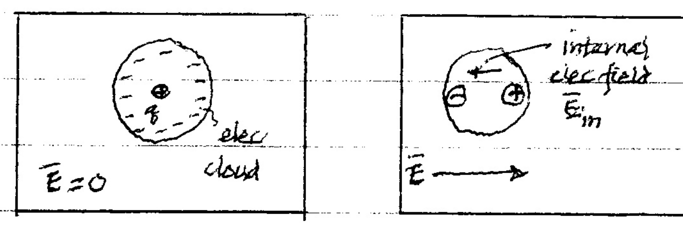
1. conductors - free charges move under influence of elec field
2. dielectrics - charges aren't free to move. they're bound. instead charges are displaced.

polar - e.g., H_2O



dipoles randomly oriented — dipoles align w/ $\bar{E}$

non-polar - e.g, oxygen, nitrogen



elec cloud shifts - effectively get dipole

effect of applying elec field to a dielectric is to create dipoles. dipoles themselves, in turn, create internal elec field $\bar{E}_{in}$

$\bar{E}_{in}$ opposite in dir to applied elec field, so total field inside dielectric (applied plus internal) is smaller than the applied elec field - applied elec field polarizes dielectric.

polarization

we quantify the effect of $\bar{E}$ on a dielectric as follows.

recall,

$$\vec{p} = Q\vec{d} = \text{dipole moment}$$

$$\vec{P} = (\text{total dipole moment}) / \text{vol}$$

$$= \text{polarization density } [C{\cdot}m/m^3 = C/m^2]$$

$$= \sum_{k=1}^{n} \vec{p}_k / \text{vol} \qquad n = \# \text{ of dipoles in vol}$$

recall elec flux density:

$$\vec{D} = \varepsilon_0 \vec{E} \quad [C/m^2]$$

in a dielectric:

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} \quad [C/m^2]$$