EE 331 - LECT. # 28 APR. 3, 2020

recall that on wednesday we found that the elec field inside a dielectric is smaller than the elec field outside it (the applied elec field). this is because the dielectric is polarized by the elec field that has been applied to it. the effect is quantified by $\bar{P}$, the polarization density.

we end our last lecture by noting that in a dielectric the elec flux density is given by:

$$\bar{D} = \varepsilon_0 \bar{E} + \bar{P} \quad [C/m^2]$$

$\varepsilon_0 \bar{E}$ = $\bar{D}$ in free space

linear, isotropic, & homogeneous dielectrics

if

$|\bar{P}| \propto |\bar{E}|$ $\propto$ = proportional to

→ $\boxed{\text{linear}}$ dielectric (opposite is nonlinear)

if

$\bar{P} \parallel \bar{E}$ $\parallel$ = parallel to

→ $\boxed{\text{isotropic}}$ (looks the same in all dirs) diel (opposite is anisotropic)

then we write:

$$\bar{P} = \chi_e \varepsilon_0 \bar{E}$$

χ_e = electric susceptibility (measure of polarizability)

thus,

$$\bar{D} = \varepsilon_0 \bar{E} + \chi_e \varepsilon_0 \bar{E} = \varepsilon_0 (1+\chi_e) \bar{E} = \varepsilon_0 \varepsilon_r \bar{E} = \varepsilon \bar{E}$$

$$\boxed{\bar{D} = \varepsilon \bar{E}} \quad [C/m^2]$$

$\varepsilon_r = 1 + \chi_e$ = relative permittivity or diel const [·]

$\varepsilon = \varepsilon_0 \varepsilon_r$ [F/m] if ε isn't a func of position,

$\rightarrow$ $\boxed{\text{homogeneous}}$ diel (opposite is inhomogeneous)

so we lump all the polarization into ε?

<u>continuity of current</u>

follows from conservation of charge - can't create or destroy charge.

start with:

$$I = \oint_S \vec{J} \cdot \vec{ds} = \text{outward flow of positive charges}$$

but inside volume amt of charge is decreasing:

$$-\frac{dQ}{dt} = \text{rate of decrease of positive charge}$$

by conservation of charge:

outward flow = rate of decrease

or

$$\oint_S \vec{J} \cdot \vec{ds} = -\frac{dQ}{dt} = -\frac{d}{dt} \int_v \rho_v dv = -\int_v \frac{\partial \rho_v}{\partial t} dv$$

why?

$$\oint_S \vec{J} \cdot \vec{ds} = \int_v \nabla \cdot \vec{J} \, dv = -\int_v \frac{\partial \rho_v}{\partial t} dv$$

how?

integrands must be equal so:

$$\boxed{\nabla \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t}}$$ eq of continuity of current

together w/ maxwell's eqs: one of the fundamental laws of EM

$\equiv$ KCL in ckt theory

<u>charge dissipation & relaxation time</u>

introduce charge into a material & over time it will dissipate.

start w/ cons. of curr:

$$\nabla \cdot \bar{J} = -\frac{\partial \rho_v}{\partial t}$$

$$\nabla \cdot \sigma \bar{E} = -\frac{\partial \rho_v}{\partial t}$$

?

ohm's law: $\bar{J} = \sigma \bar{E}$

next,

$$\nabla \cdot \bar{E} = -\frac{1}{\sigma}\frac{\partial \rho_v}{\partial t}$$

$$\rho_v/\varepsilon = -\frac{1}{\sigma}\frac{\partial \rho_v}{\partial t}$$

?

gauss's law/m's first: $\nabla \cdot \bar{D} = \rho_v$

rearranging:

$$\frac{\partial \rho_v}{\partial t} + \frac{\sigma}{\varepsilon}\rho_v = 0 \qquad ?$$

first-order ode ∴ soln:

$$\rho_v(t) = \rho_{v_0} e^{-(\sigma/\varepsilon)t} = \rho_{v_0} e^{-t/T_r}$$ - charge decays exponentially!

where

ρ_{v_0} = initial vol charge density at $t=0$

$T_r = \varepsilon/\sigma$ [s] = relaxation time or rearrangement time

$T_r = t \rightarrow \rho_v = e^{-1}\rho_{v_0}$ 36.8% of init value

good cond: $\sigma > \varepsilon \rightarrow T_r$ small e.g., cu

good diel: $\sigma < \varepsilon \rightarrow T_r$ large e.g., fused quartz

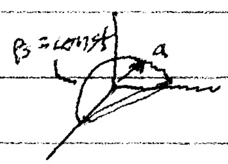
Cu: $T_r = 1.53 \times 10^{-19}$ s

fused quartz: $T_r \cong 51$ days $>$ 4 million seconds

example #21

potential example

ρ_s = const on surf of hemisphere of radius = a, find the potential at the origin due to this charge.



$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho_s \, ds'}{|\bar{r}-\bar{r}'|} = \frac{\rho_s}{4\pi\varepsilon_0}\int_0^{2\pi} d\phi' \int_0^{\pi/2} \frac{a^2 \sin\theta' \, d\theta'}{a}$$

$ds' = r'^2 \sin\theta' \, d\theta' \, d\phi'$

$= a^2 \sin\theta' \, d\theta' \, d\phi'$

$$= \frac{\rho_s a}{4\pi\varepsilon_0} \phi' \Big|_0^{2\pi} \cos\theta' \Big|_{\pi/2}^{0}$$

$\bar{r} = 0$

$\bar{r}' = r'\hat{a}_r = a\hat{a}_r$ m

$$= \frac{\rho_s a}{4\pi\varepsilon_0}(2\pi)(1) = \boxed{\frac{\rho_s a}{2\varepsilon_0} \text{ V.}}$$

$|\bar{r}-\bar{r}'| = a$ m

limits $\int_0^{2\pi} d\phi' \int_0^{\pi/2} d\theta'$