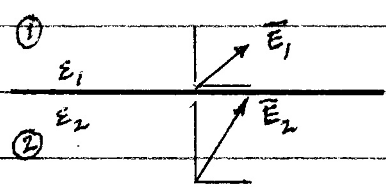
EE 331 - LECT. #29 APR. 6, 2020

boundary conditions (BC)

- fields inside and outside a medium differ
- relationship btwn fields at interface determined by boundary conditions
- true for any shaped boundary and at every point
- true for both electrostatics and electromagnetics
- BC relate tangential components & normal components, come from $\oint_S \bar{D}\cdot d\bar{s} = Q$ and $\oint_L \bar{E}\cdot d\bar{l} = 0$

diel-diel BC (two diff dielectrics)



$\bar{E}_1 = \bar{E}_{1t} + \bar{E}_{1n}$

= tangential vector component + normal vector component

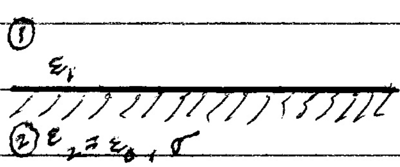
$\bar{E}_2 = \bar{E}_{2t} + \bar{E}_{2n}$

BC: 1) $E_{1t} = E_{2t}$ → tangential components of $\bar{E}$ continuous

2) $D_{1n} - D_{2n} = \rho_s$ → normal " " not continuous ($\bar{D} = \varepsilon\bar{E}$)

$D_{1n} = D_{2n}$ for $\rho_s = 0$ → if ρ_s not given, assume it's 0

diel-cond BC



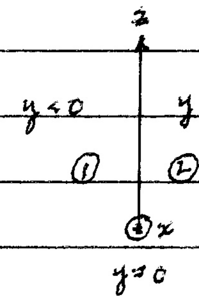
BC: 1) $E_{1t} = 0$

2) $D_{1n} = \rho_s$

in perfect cond, $\sigma \to \infty$, $\bar{E} = 0$ because V is const ($\bar{E} = -\nabla V$)

<u>normal & tangential components</u> (equivalent to perpendicular & parallel for flat surf)

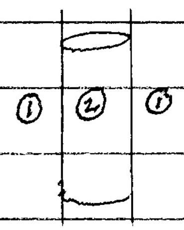
example #1



norm ?

tang ?

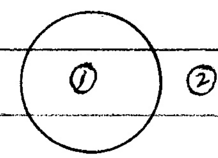
example #2



norm ?

tang ?

example #3



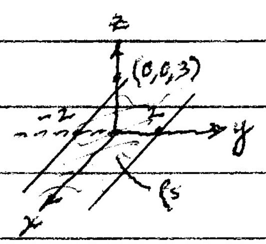
norm ?

tang ?

examples #22 & #23

<u>C's law example</u>

strip of charge in $z=0$ plane with $\rho_s = y^2\ nC/m^2$. strip infinite in $\pm x$ directions & extending from $y=2$ to $y=2\,m$. find $\bar{E}$ @ $P(5,0,3)$.



since strip infinitely long in x dir, $\bar{E}(5,0,3) = \bar{E}(0,0,3)$

$$\bar{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho_s(\bar{r}-\bar{r}')}{|\bar{r}-\bar{r}'|^3}\, ds'$$

$\rho_s = y'^2\ nC/m^2$

$\bar{r} = (0,0,3)\ m$

$\bar{r}' = (x',y',0)\ m$

$\bar{r}-\bar{r}' = (-x',-y',3)\ m$

$|\bar{r}-\bar{r}'|^3 = [x'^2+y'^2+9]^{3/2}\ m^3$

$ds' = dx'dy'\ m^2$

$\int_{-2}^{2} dy' \int_{-\infty}^{\infty} dx'$

$$\bar{E} = \frac{10^{-9}}{4\pi\varepsilon_0}\int_{-2}^{2} dy' \int_{-\infty}^{\infty} \frac{y'^2(\overset{0}{\cancel{-x'}}, \overset{0}{\cancel{-y'}}, 3)}{[x'^2+y'^2+9]^{3/2}}\, dx'$$

by symmetry

$$= \frac{3\times10^{-9}}{\pi\varepsilon_0}\hat{z}\int_0^2 y'^2 dy' \int_0^{\infty} \frac{dx'}{[x'^2+\underbrace{y'^2+9}_{a^2}]^{3/2}}$$

even funcs

$$\vec{E} = \frac{3\times 10^{-9}}{\pi\varepsilon_0}\hat{a}_z \int_0^2 y'^2\,dy' \left.\frac{x'}{(y'^2+9)\sqrt{x'^2+y'^2+9}}\right|_0^{\infty} = \frac{3\times 10^{-9}}{\pi\varepsilon_0}\hat{a}_z \int_0^2 \frac{y'^2}{y'^2+9}\,dy'$$

$$= \frac{3\times 10^{-9}}{\pi\varepsilon_0}\hat{a}_z \left(y' - 3\tan^{-1}(y'/3)\right)\Big|_0^2 = \frac{3\times 10^{-9}}{\pi\varepsilon_0}(0.2360)\hat{a}_z$$

$$= \boxed{25.4533\,\hat{a}_z \quad V/m}$$