EE 331 – LECT. #30 APR. 8, 2020

capacitance

for 2 conductors separated by a dielectric:

$$C = Q/V \quad \text{ratio always constant}$$

where

Q = charge on one conductor (>0)

V = pot diff btwn 2 conductors (>0)

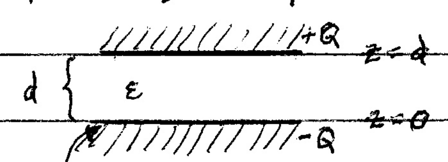
$\boxed{\text{cap is func only of physical properties}}$ – increasing Q won't help

recipe for finding capacitance:

1. make sketch and choose coord system
2. set Q & $-Q$ on conductors
3. find $\overline{E}$
4. find V from $\overline{E}$ $\left(V = -\int \overline{E}\cdot\overline{dl}\right)$
5. use V in defn of cap $(C = Q/V)$

examples:

[1] parallel plate cap



S = surf area

$\sqrt{S} >> d$

1-2. cart coords, Q

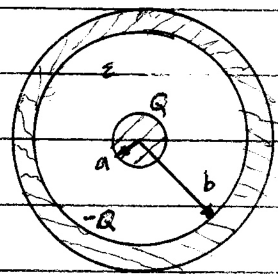
3. $\overline{E}$ from BC:

$$D_n = \rho_s = Q/S \rightarrow E_n = \frac{D_n}{\varepsilon} = \frac{Q}{\varepsilon S}$$

4. $$V = -\int_0^d \overline{E}\cdot\overline{dl} = -\int_0^d -\frac{Q}{\varepsilon S}\hat{a}_z \cdot dz\,\hat{a}_z = \frac{Qd}{\varepsilon S}$$

5. $$C = Q/V = -Q\Big/\frac{Qd}{\varepsilon S} = \boxed{\frac{\varepsilon S}{d}\ [F]}$$

2 spherical cap



$b \gg a$

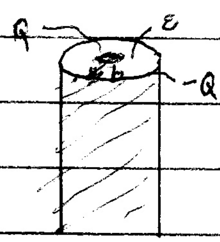
1.-2. spherical coords, Q

3. $\bar{E} = \frac{Q}{4\pi\varepsilon r^2}\hat{a}_r$

4. $V = -\int \bar{E}\cdot d\bar{l} = -\int_b^a \frac{Q}{4\pi\varepsilon r^2}\hat{a}_r \cdot dr\,\hat{a}_r = \frac{Q}{4\pi\varepsilon}\left(\frac{1}{a}-\frac{1}{b}\right)$

5. $C = Q/V = \boxed{\frac{4\pi\varepsilon}{\left(\frac{1}{a}-\frac{1}{b}\right)}\ [F]}$

3 coaxial cap



$b \gg a$

1.-2. cyl coords, Q

3. $\bar{E} = \frac{Q}{2\pi\varepsilon\rho L}\hat{a}_\rho$

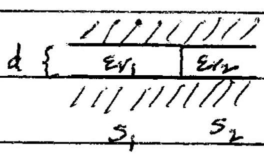
4. $V = -\int \bar{E}\cdot d\bar{l} = -\int_b^a \frac{Q}{2\pi\varepsilon L\rho}\hat{a}_\rho \cdot d\rho\,\hat{a}_\rho = \frac{Q}{2\pi\varepsilon L}\ln(b/a)$

5. $C = Q/V = \boxed{\frac{2\pi\varepsilon L}{\ln(b/a)}\ [F]}$

how can C be increased?

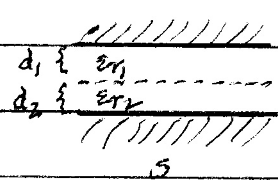
parallel and series capacitors

parallel



$C = C_1 + C_2 \qquad C_1 = \frac{\varepsilon_1 S_1}{d}, \quad C_2 = \frac{\varepsilon_2 S_2}{d}$

series



$C = \frac{C_1 C_2}{C_1 + C_2} \qquad C_1 = \frac{\varepsilon_1 S}{d_1}, \quad C_2 = \frac{\varepsilon_2 S}{d_2}$

supercapacitors (ultracapacitors) - up to at least 12,000 F voltage breakdown

when voltage too large, electrons no longer displaced in dielectric but actually torn away & curr will flow btwn conductors. dielectric strength is characteristic of a dielectric. when voltage breakdown occurs, end up w/ a fried capacitor.

french guys' equations - diffeqs

recall g's law (m's first):

$$\nabla \cdot \bar{D} = \rho_v \rightarrow \nabla \cdot \bar{E} = \rho_v/\varepsilon \quad (\bar{D} = \varepsilon \bar{E})$$

also recall,

$$\bar{E} = -\nabla V$$

thus,

$$\nabla \cdot \bar{E} = \nabla \cdot (-\nabla V) = \rho_v/\varepsilon$$

$$\rightarrow \boxed{\nabla^2 V = -\rho_v/\varepsilon}$$ poisson's eq

in a charge-free region:

$$\boxed{\nabla^2 V = 0}$$ laplace's eq

solve p's or l's eq to find V.

uniqueness thm - proof by contradiction

→ if you find a soln to l's or p's eq that satisfies the values at the boundaries, the soln is unique, i.e. it's the only soln.

notes:

1. don't confuse values at boundaries w/ boundary conditions (BC); they're not the same!
2. it doesn't matter how you find a soln - you can even just guess
3. in order to show a soln is correct, must do two things: show that it satisfies l's or p's eq by plugging it into the diffeq & then show that it satisfies the values at the boundaries. finally, invoke the uniqueness thm.