EE 331, LECT. #31 APR. 10, 2020

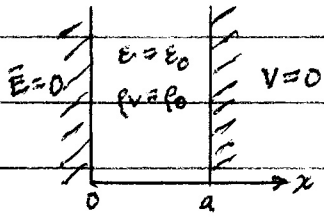
boundary value problems - solving p's or l's eq for V

once we know V, we can find the capacitance. can even find V using numerical methods (ch. 14).

recipe for finding capacitance:

1. use V to find $\bar{E}$
2. use $\bar{E}$ to find $\bar{D}$
3. use $\bar{D}$ to find ρ_s (BC)
4. use ρ_s to find Q
5. use Q to find C_{cap} ($C = Q/V$)

example p's eq.



p's eq: $\nabla^2 V = -\rho_v/\varepsilon \rightarrow \frac{d^2V}{dx^2} = -\rho_0/\varepsilon_0$

integ once wrt x:

$$\frac{dV}{dx} = \frac{-\rho_0}{\varepsilon_0} x + A \qquad (1)$$

integ again:

$$V(x) = \frac{-\rho_0}{2\varepsilon_0} x^2 + Ax + B \qquad (2)$$

apply BV:

$$\bar{E} = -\frac{dV}{dx} \hat{a}_x \quad (\text{recall } \bar{E} = -\nabla V)$$

@ $x = 0$: $\bar{E}(0) = \frac{\rho_0}{\varepsilon_0}(0) + A = 0 \rightarrow A = 0$ from (1)

@ $x = a$: $V(a) = \frac{-\rho_0}{2\varepsilon_0} a^2 + B = 0 \rightarrow B = \frac{\rho_0 a^2}{2\varepsilon_0}$ from (2)

$$\therefore \boxed{V(x) = \frac{\rho_0}{2\varepsilon_0}(a^2 - x^2)}$$

example #24 - 1's eq

example #25 - 2's eq

1's eq - 2D (only cart coords)

$$\nabla^2 V = 0 \rightarrow \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

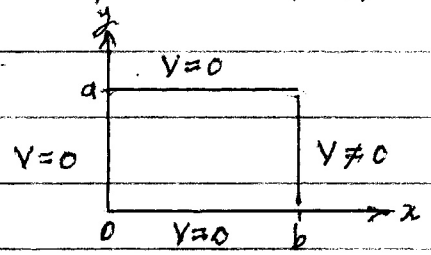
use separation of variables to solve:

$$V(x,y) = X(x)Y(y) \text{ - solve for } X(x) \& Y(y) \text{ independently}$$

end up with:

$$V(x,y) = \underbrace{(A\sinh\beta x + B\cosh\beta x)}_{X(x)}\underbrace{(C\sin\beta y + D\cos\beta y)}_{Y(y)}$$

use 4 BV to find consts and β. we must specify BV. we'll use:



$V(0,y) = 0$

$V(x,0) = 0$

$V(x,a) = 0$

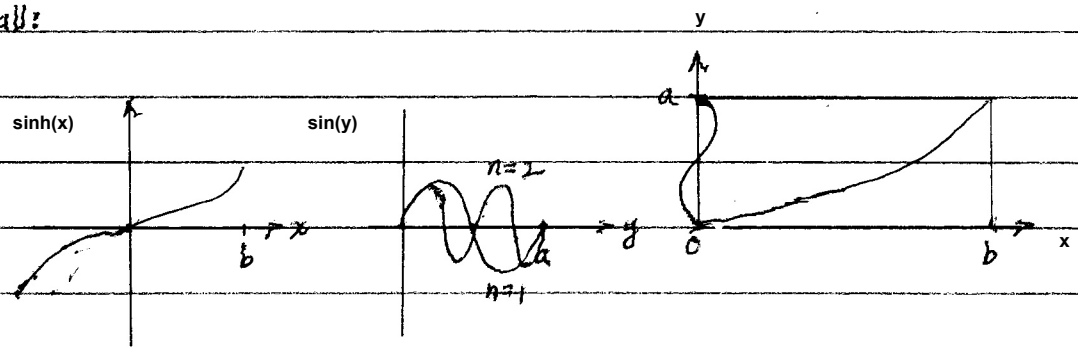
$V(b,y) \neq 0$

using BV in (1) gives:

$$V(x,y) = \sum_{n=1}^{\infty} C_n \sinh\left(\frac{n\pi}{a}x\right)\sin\left(\frac{n\pi}{a}y\right) \quad [V]$$

find C_n's using 4th BV - $V = V(b,y)$

recall:



example #26