EE 331, LECT. #23 APR. 15, 2020

magnetic flux & magnetic flux density

recall, for electrostatics:

$$\vec{E} \rightarrow \vec{D} = \varepsilon\vec{E} = \text{elec flux density } [C/m^2]$$

similarly, for magnetostatics:

$$\vec{H} \rightarrow \vec{B} = \mu\vec{H} = \text{magn flux density } [Wb/m^2],\ [T],\ [G]$$

where:

Wb = weber

T = tesla

G = gauss

$\varepsilon = \varepsilon_r \varepsilon_0$ = permittivity $[F/m]$ → capacitance

$\mu = \mu_r \mu_0$ = permeability $[H/m]$ → inductance

μ_r = relative permeability $[\cdot]$

μ_0 = free-space permeability $= 4\pi \times 10^{-7}$ $[H/m]$ = constant

also, recall:

$$\psi = \int_S \vec{D} \cdot \vec{ds} = \text{elec flux } [C]$$

for magnetostatics:

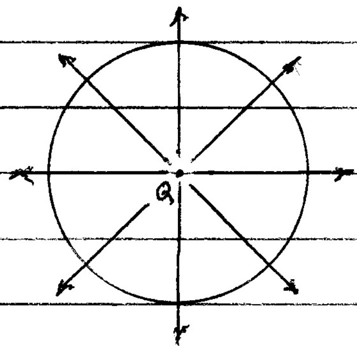
$$\psi = \int_S \vec{B} \cdot \vec{ds} = \text{magn flux } [Wb]$$

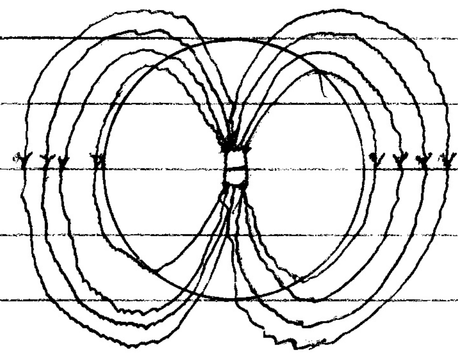
next, recall:

$$\oint_S \bar{D} \cdot \bar{ds} = Q_{enc}$$

for magnetostatics:

$$\oint_S \bar{B} \cdot \bar{ds} = 0 \longrightarrow \text{no magnetic monopoles}$$





next,

$$\oint_S \bar{B} \cdot \bar{ds} \overset{?}{=} \int_v \nabla \cdot \bar{B} \, dv = 0$$

thus,

$$\boxed{\nabla \cdot \bar{B} = 0} \quad \text{no magn monopoles (g's law for magn fields)}$$

maxwell's eqs for static fields

$$\left.\begin{array}{lll} \nabla \cdot \bar{D} = \rho_v & \text{g's law} \\ \nabla \cdot \bar{B} = 0 & \text{no magn monopoles} \end{array}\right\} \text{final form}$$

$$\left.\begin{array}{lll} \nabla \times \bar{E} = 0 & \bar{E} \text{ conservative} \\ \nabla \times \bar{H} = \bar{J} & \text{amp's law} \end{array}\right\} \begin{array}{l} \text{change for time-varying fields} \\ (\partial/\partial t \text{ terms added}) \end{array}$$

example: $\vec{A} = y\cos ax\,\hat{a}_x + (y + e^{-x})\hat{a}_z$

valid $\vec{B}$ field? valid $\vec{E}$ field?

$$\nabla\cdot\vec{B} = 0 \quad \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} = \frac{\partial}{\partial x}(y\cos ax) + 0 + \frac{\partial}{\partial z}(y + e^{-x})$$

$$= -ay\sin ax + 0 \neq 0$$

$\rightarrow \hat{A}$ can't be magn field

$$\nabla\times\vec{E} = 0 \quad \left[\frac{\partial A_z}{\partial y} - \cancel{\frac{\partial A_y}{\partial z}}\right]\hat{a}_x + \left[\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right]\hat{a}_y + \left[\cancel{\frac{\partial A_y}{\partial x}} - \frac{\partial A_x}{\partial y}\right]\hat{a}_z$$

$$\frac{\partial}{\partial y}(y + e^{-x})\hat{a}_x + \left[\frac{\partial}{\partial z}(y\cos ax) - \frac{\partial}{\partial x}(y + e^{-x})\right]\hat{a}_y - \frac{\partial}{\partial y}(y\cos ax)\hat{a}_z$$

$$= \hat{a}_x + e^{-x}\hat{a}_y - \cos ax\,\hat{a}_z \neq 0$$

$\rightarrow \hat{A}$ can't be elec field