EE 331 - LECT. 36 APR. 24, 2020

example #28

force on curr element - $\vec{E} = 0$, $\vec{B} \neq 0$

for teeny elem of moving charge:

$$d\vec{F} = dQ\,\vec{u} \times \vec{B}$$
$$= dQ\,\frac{d\vec{l}}{dt} \times \vec{B}$$
$$= \frac{dQ}{dt}\,d\vec{l} \times \vec{B}$$
$$d\vec{F} = I\,d\vec{l} \times \vec{B}$$

$\uparrow I d\vec{l}$ $\odot \vec{B}$ from other src

add up force on each little elem of curr:

$$\vec{F} = \int I\,d\vec{l} \times \vec{B}$$ = force on line curr I due to $\vec{B}$ from other source

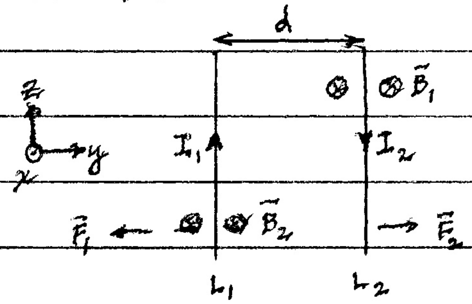
special case - uniform $\vec{B}$ & straight cond L

$$\vec{F} = I\vec{L} \times \vec{B} \quad [N]$$
= force on straight cond in uniform magn field

force btwn straight parallel conductors

consider two straight parallel conductors:

currents in opp dirs



I_1 creates $\vec{B}_1 \rightarrow \vec{B}_1$ exerts force on I_2 in L_2

I_2 creates $\vec{B}_2 \rightarrow \vec{B}_2$ exerts force on I_1 in L_1

Q: what dir is force on L_1 due to $\vec{B}_2$? A: $\vec{F}_1 = I_1 \vec{L}_1 \times \vec{B}_2$ $-\hat{a}_y$ by RHR
($\vec{F}_1$ ← force on L_1)

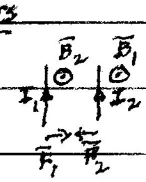
Q: what dir is force on L_2 due to $\vec{B}_1$? A: $\vec{F}_2 = I_2 \vec{L}_2 \times \vec{B}_1$ $\hat{a}_y$ by RHR

∴ curr in opp dirs → wires repel each other

magnitude of force: $F_1 = I_1 L_1 B_2 = I_1 \mu_1 \frac{\mu_0 I_2}{2\pi\rho} L_1 = \mu_0 \frac{I_1 I_2}{2\pi\rho} L_1$ $\quad \vec{H}_{wire} = \frac{I}{2\pi\rho}\hat{a}_\phi$

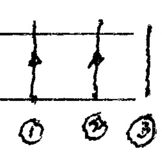
$$\rightarrow F_1/L_1 = F_2/L_2 = \mu_0 \frac{I_1 I_2}{2\pi d} \quad [N/m]$$

currents in same dirs



∴ curr in same dirs → wires attract each other

example:



$\bar{F}_1 = 0$ $\quad I_3$ up or down

$\bar{F}_2 = 0$ $\quad I_3$ up or down

magnetic dipole - analog of elec dipole

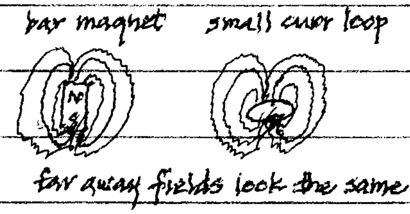
recall,

$$\vec{E} = \frac{p}{4\pi\varepsilon r^3}(2\cos\theta\,\hat{a}_r + \sin\theta\,\hat{a}_\theta) \; [V/m]$$

$\vec{p} = Q\vec{d}$ [C-m] = dipole moment   Q, -Q } d

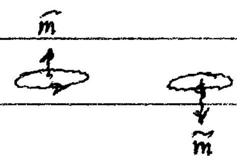
for magnetostatics:

$$\vec{H} = \frac{m}{4\pi r^3}(2\cos\theta\,\hat{a}_r + \sin\theta\,\hat{a}_\theta) \; [A/m]$$



$\vec{m} = IS\,\hat{a}_n$ [A-m²] = magnetic moment

- S = surf area of loop
- dir of $\vec{m}$ given by rhr: fingers in dir of curr, thumb gives dir of $\vec{m}$



video