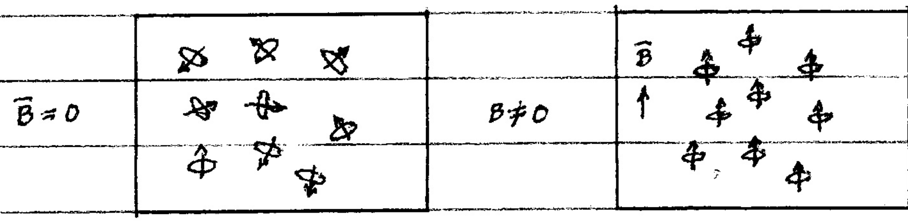
EE331 - LECT. #37 APR. 27, 2020

## magnetic fields in materials

magnetization & permeability - analogs to polarization & permittivity

use simple atom model of electrons orbiting a nucleus




randomly oriented dipoles $\bar{B}$ aligns dipoles

get magnetization:

$\bar{M} = n\bar{m} = $ magn moment/vol [A/m]

$n =$ # atoms/vol

$\bar{m} =$ magn moment [$A\text{-}m^2$] (recall, $\bar{m} = IS\hat{a}_n$)

material classification

non-magnetic (linear): $\mu_r \cong 1$

$$\bar{B} = \mu_0(\bar{H} + \bar{M}) = \mu_0(\bar{H} + \chi_m\bar{H}) = \mu_0(1+\chi_m)\bar{H} = \mu_0\mu_r\bar{H}$$

or

$$\bar{B} = \mu\bar{H}$$

with,

$\mu_r = 1 + \chi_m$ [·] (analog of $\epsilon_r$)

$\chi_m =$ magnetic susceptibity [·] (analog of $\chi_e$)

most materials in ee331, diels & conds, are linear

diamagnetic: $\chi_m < 0$, $\mu_r \cong 1$ e.g., copper

paramagnetic: $\chi_m > 0$, $\mu_r \cong 1$ e.g., tungsten

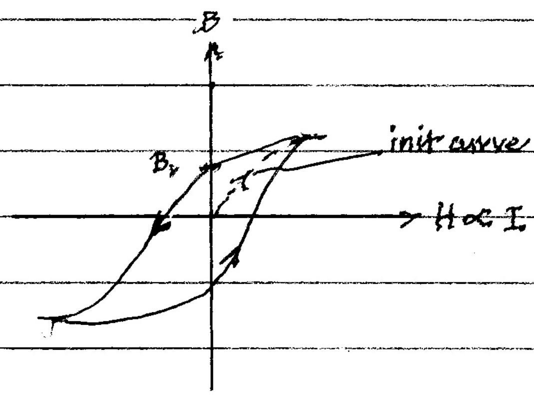
note: perfect diamagnetic $\chi_m = -1$, $\mu_r = 0$, $\bar{B} = 0$ → superconductor

in statics, $\bar{E} = 0$ in conductor but $\bar{B} \neq 0$. however, $\bar{B} = 0$ in superconductor so don't get loss in superconductor.

magnetic (nonlinear): $\mu_r \gg 1$

ferromagnetic: $\chi_m \gg 0$, $\mu_r \gg 1$ e.g., iron $\mu_r \approx 3000$

B-H curve




- hysteresis loop, B lags H
- $B_r$ = permanent flux density (permanent magnets)
- area of loop gives energy loss per unit vol → heat loss in material, minimize

materials in ee 361 & 362 are nonlinear

boundary conditions

① $\mu_{r_1}$ ⊙ $K$

② $\mu_{r_2}$

BC: 1) $H_{1t} - H_{2t} = K \rightarrow H_{1t} = H_{2t}$ for $K = 0$

2) $B_{1n} = B_{2n}$

where $K$ = surf curr density [A/m]

also use $\bar{B} = \mu \bar{H}$

notes: 1) same BC for conductors & dielectrics

2) elec field BC "opposite" to magn field BC

$$E_{1t} = E_{2t} \qquad H_{1t} - H_{2t} = K$$

$$D_{1n} - D_{2n} = \rho_s \qquad B_{1n} = B_{2n}$$

inductors & inductance

recall,

$$C = Q/V \qquad \text{[F]} \ [\cancel{C}]$$

similarly,

$$L = \lambda / I \qquad [H]$$

where

$$\lambda = N \int \bar{B} \cdot d\bar{s} = N\psi = \text{flux linkage}$$

$N$ = number of turns of wire

inductance func only of physical properties of inductor

common (self) inductances (table 8.3)

coaxial cable: $\boxed{L = \frac{\mu_0 l}{2\pi} \ln(b/a) \quad [H]}$

$l$ = length [m]

$b > a$

solenoid: $\boxed{L = \frac{\mu_0 N^2 S}{l} \quad [H]}$

$l$ = length [m]

$S$ = area of cross section [$m^2$]

$a$ = radius of solenoid [m]

$N$ = # of turns of wire

$l \gg a$

example #29