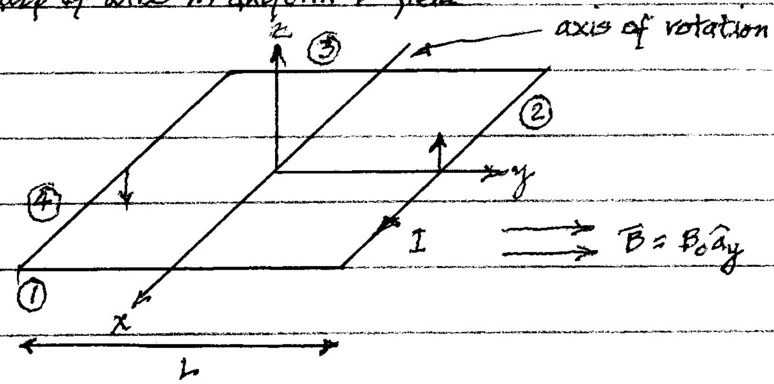
EE 331 – LECT. #38 APR. 29, 2020

please do student evals: mywsu → manage classes tile → course evals tab

final exam in one week, 5.6.20 via zoom & blackboard, 1-3 pm but otherwise same as exam #3; review session 5.5.20, 6-7 pm via zoom

magnetic torque

place square loop of wire in uniform $\bar{B}$ field




$\bar{F} = I\bar{L} \times \bar{B}$

forces on sides:

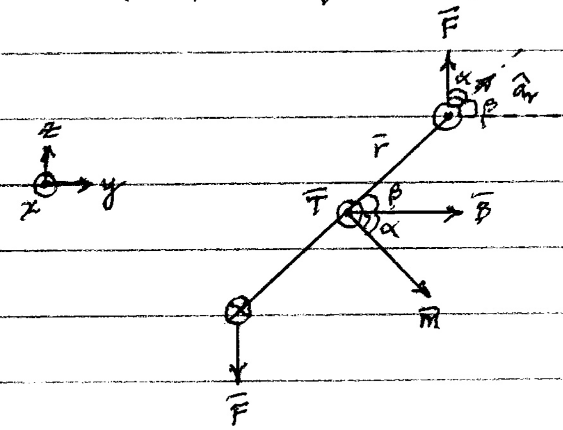
$\bar{F}_1 = 0$, $\bar{F}_3 = 0$ } $\bar{B}$ parallel to L

$\bar{F}_2 = ILB_0\,\hat{a}_z$, $\bar{F}_4 = -ILB_0\,\hat{a}_z$ } forces in opposite dirs cause loop to rotate around x-axis

→ get torque

calculating torque

consider side view of loop at angle in $\bar{B}$ field:




recall,

$$\bar{T} = \bar{r} \times \bar{F} \quad [N\text{-}m]$$

$\bar{r}$ = moment arm

for half of loop length:

$$\bar{T}_1 = |\bar{r}||\bar{F}| \sin\alpha \, \hat{a}_n \qquad \hat{a}_n \text{ perpendicular to page}$$

$$= (L/2)(IB_oL) \sin\alpha \, \hat{a}_n$$

for whole loop:

$$\bar{T} = 2\bar{T}_1 = IB_oL^2 \sin\alpha \, \hat{a}_n$$

but

$$L^2 = \text{area of loop} = S$$

and

$$\bar{m} = IS\hat{a}_n = \text{magn moment}$$

thus,

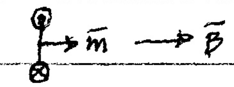
$$\bar{T} = (IL^2) B_o \sin\alpha \, \hat{a}_n = ISB_o \sin\alpha \, \hat{a}_n = |\bar{m}| B_o \sin\alpha \, \hat{a}_n$$

but

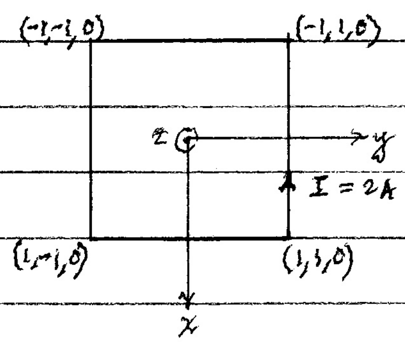
$\alpha$ = angle btwn $\bar{m}$ & $\bar{B}$

and finally,

$$\bar{T} = \bar{m} \times \bar{B} \quad \{N\text{-}m\}$$

→ torque wants to align $\bar{m}$ & $\bar{B}$  $\bar{m} \times \bar{B} = 0$

example




for $\bar{B} = 0.5\,\hat{a}_x$ Wb/m, find $\bar{T}$ & axis of rotation

$\bar{m} = IS\,\hat{a}_n = (2)(4)\,\hat{a}_z = 8\,\hat{a}_z \rightarrow \bar{T} = \bar{m} \times \bar{B} = 8\hat{a}_z \times 0.5\hat{a}_x = 4\,\hat{a}_y \quad \{N\text{-}m\}$

y = axis of rotation

magnetic energy

recall,

$$W_e = \frac{1}{2} C V^2 \; \{J\} = \text{energy stored in cap}$$

in general,

$$W_e = \frac{1}{2}\int_v \varepsilon |\bar{E}|^2 dv \; \{J\} = \text{elec energy stored in material}$$

also,

$$w_e = \frac{1}{2}\varepsilon |\bar{E}|^2 \; \{J/m^3\} = \text{energy density in material}$$

similarly,

$$W_m = \frac{1}{2} L I^2 \ [J] = \text{energy stored in inductor},$$

$$W_m = \frac{1}{2} \int_v \mu |\bar{H}|^2 dv = \text{magn energy stored in material} \ [J]$$

$$w_m = \frac{1}{2} \mu |\bar{H}|^2 \ [J/m^3] = \text{energy density in material.}$$