EE 331 - LECT. #3 JAN. 17, 2020

phasor conversion

$$V(z,t) = \underbrace{5e^{-0.2z}}_{\text{magn}} \cos(\omega t - \underbrace{0.3}_{\text{phase}})$$

go from time to phasor by inspection:

$$V_s(z) = 5e^{-0.2z} e^{-j0.3}$$

go from phasor to time:

$$V(z,t) = \text{Re}\{V_s(z) e^{j\omega t}\}$$

$$= \text{Re}\{5e^{-0.2z} e^{-j0.3} e^{j\omega t}\}$$

$$= \text{Re}\{5e^{-0.2z} e^{j(\omega t - 0.3)}\}$$

$$= 5e^{-0.2z} \text{Re}\{e^{j(\omega t - 0.3)}\}$$ ← use euler's rule

$$= 5e^{-0.2z} \text{Re}\{\cos(\omega t - 0.3) + j\sin(\omega t - 0.3)\}$$

$$= 5e^{-0.2z} \cos(\omega t - 0.3)$$

traveling waves

start with scalar wave eq:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{u^2}\frac{\partial^2 y}{\partial t^2}$$ ← 2nd order pde w/ variables x, t

y = some field

general soln of wave eq is given by:

$$y(x,t) = y^+(t - x/u) + y^-(t + x/u)$$

wave traveling in $+x$ dir @ speed u — wave traveling in $-x$ dir @ speed u

how do we know $y(x,t)$ is the soln? use in both sides of pde. if LHS = RHS, then we've shown it's a soln. "proof" on notes page.

we'll only consider sinusoidal (time-harmonic) waves of the form:

$$y(x,t) = A\cos[\omega(t - x/u)] + B\cos[\omega(t + x/u)]$$

$$= A\cos(\omega t - \beta x) + B\cos(\omega t + \beta x)$$

= (wave in $+x$ dir) + (wave in $-x$ dir) definitions:

1. $u = \omega/\beta$ = wave or phase velocity [m/s]

2. $\beta = \omega/u$ = phase const [rad/m]

3. $\omega = 2\pi f = \frac{2\pi}{T}$ = radian freq [rad/s]

4. f = freq [Hz]

5. $T = 1/f$ = period [s]

6. $\lambda = 2\pi/\beta$ = wavelength [m]

if $y^+(x,t) = A\cos(\omega t - \beta x)$, what is max ampl of wave?

7. $y^+_{max} = A$

if $y^+(x,t)$ propagates in lossy material, ampl will decrease.

$$y^+(x,t) = Ae^{-\alpha x}\cos(\omega t - \beta x)$$

8. α = attenuation const [Np/m]