EE 331 ~ LECT. #4 JAN. 22, 2020

start w/ scalar wave eq:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{u^2}\frac{\partial^2 y}{\partial t^2}$$

soln to this can be time-harmonic wave.

$$y(x,t) = A\cos(\omega t - \beta x) + B\cos(\omega t + \beta x)$$

wave in +x dir   wave in −x dir

$$y(x,t) = Ae^{-\alpha x}\cos(\omega t - \beta x) + Be^{\alpha x}\cos(\omega t + \beta x)$$

α = attenuation const [Np/m]

β = phase const [rad/m]

don't want to deal w/ both time & position at the same time so we transform quantities into freq domain. recall,

$T = 1/f$ → short period T ⟷ high freq f

long period T ⟷ low freq f

also,

$\frac{d}{dt}$ ⟷ multiplication in freq domain

$\int dt$ ⟷ division in freq domain

complete form of time-harmonic traveling wave:

$$V_s(z) = V_0^+ e^{-\alpha z} e^{-j(\beta z + \phi_0^+)}$$

$$+ V_0^- e^{\alpha z} e^{+j(\beta z + \phi_0^-)}$$

rewrite as:

$$\boxed{V_s(z) = V_0^+ e^{-\gamma z + j\phi_0^+} + V_0^- e^{\gamma z + j\phi_0^-}}$$

$\gamma = \alpha + j\beta$. [1/m] = propagation const

γ is const for a given freq.

ϕ_0^+, ϕ_0^- ~ phase angles [rad]

$u = \omega/\beta$ = wave velocity [m/s]

in time domain:

$$V(z,t) = V_0^+ e^{-\alpha z} \cos(\omega t - \beta z + \phi_0^+)$$

$$+ V_0^- e^{\alpha z} \cos(\omega t + \beta z + \phi_0^-)$$