

in last lecture:

$$\frac{\ell}{\lambda} \ll 1 \rightarrow \text{ignore TL}$$

$$\frac{\ell}{\lambda} \gtrsim 0.01 \rightarrow \text{may not be able to ignore}$$

for $\ell = 2.5 \text{ cm}$, $f = 2 \text{ kHz}$:

$$\frac{\ell}{\lambda} \approx 1.7 \times 10^{-7} \ll 1$$

for $\ell = 10 \text{ km}$, $f = 2 \text{ kHz}$:

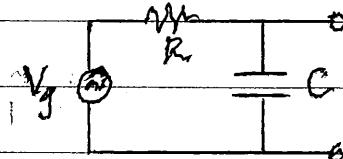
$$\frac{\ell}{\lambda} \approx 0.07 \gtrsim 0.01$$

power in U.S. $\rightarrow f = 60 \text{ Hz}$, for $v = c$ what is λ ?

$$\lambda = \frac{v}{f} = 5 \times 10^6 \text{ m} \approx 3107 \text{ mi}$$

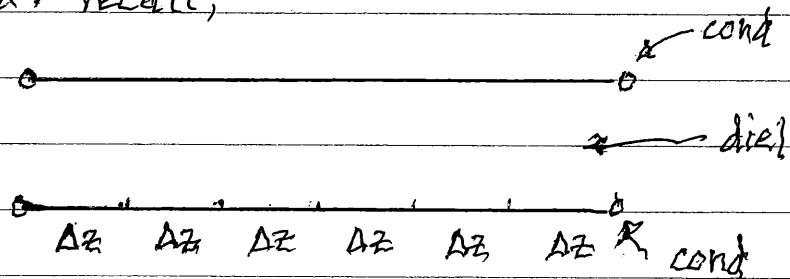
Lumped element model of TL

consider:

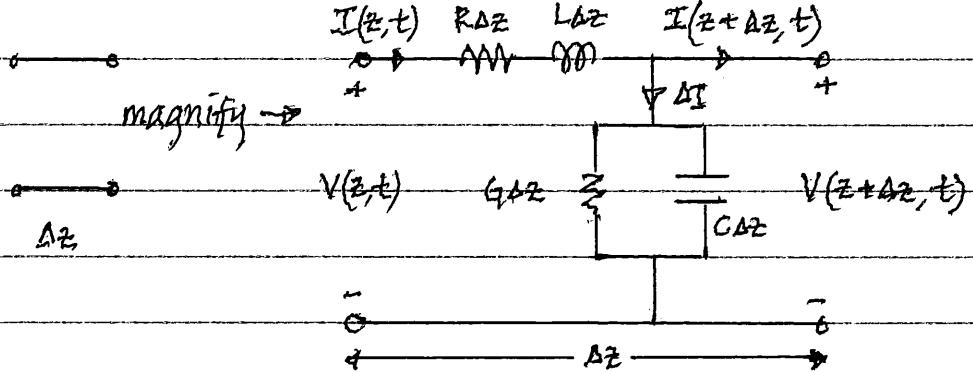


$R, C, \text{ etc.}$ are
lumped elements

TL's have $R, C, \text{ etc.}$, but distributed throughout TL,
not lumped. recall,



we break the line into tiny chunks Δz long.
consider one chunk:



Lumped element model

TL parameters

1. R, L, G, C
2. funcs only of physical properties of a TL,
i.e., the way they're made \Rightarrow for a fixed f
(table II-1)
3. units in per meter

e.g., $R \left[\Omega/m \right] R\Delta z = \left[\Omega/m \right] [m] = [\Omega]$

4. 2 conductors $\rightarrow R, L$

- R = resistance (loss in conductors) $\left[\Omega/m \right]$

- L = inductance $\left[H/m \right]$

5. dielectric separating conductors $\rightarrow G$

- G = conductance (loss in dielectric) $\left[S/m \right]$

6. 2 cond + die $\rightarrow C$

- C = capacitance $\left[F/m \right]$

derivation of TL eqs - want to find $\frac{\partial V}{\partial z} I$

$$\text{KVL: } V(z, t) = R_{AB} I(z, t) + L_{AB} \frac{\partial I(z, t)}{\partial t} + V(z + \Delta z, t)$$

rearrange & take limit as $\Delta z \rightarrow 0$

$$\lim_{\Delta z \rightarrow 0} \frac{V(z + \Delta z, t) - V(z, t)}{\Delta z} = R I(z, t) + L \frac{\partial I(z, t)}{\partial t}$$

$$\left[-\frac{\partial V(z, t)}{\partial z} = R I(z, t) + L \frac{\partial I(z, t)}{\partial t} \right]$$

$$\text{KCL: } \frac{I(z, t) - I(z + \Delta z, t)}{\Delta z}$$

$$I(z, t) = \alpha I + I(z + \Delta z, t)$$

$$\begin{aligned} &= Q_{AB} V(z + \Delta z, t) + C_{AB} \frac{\partial V(z + \Delta z, t)}{\partial t} \\ &\quad + \beta(z + \Delta z, t) \end{aligned}$$

rearrange & take limit:

$$\lim_{\Delta z \rightarrow 0} \frac{I(z + \Delta z, t) - I(z, t)}{\Delta z} \approx Q V(z + \Delta z, t)$$

$$+ C \frac{\partial V(z + \Delta z, t)}{\partial t}$$

$$\left[-\frac{\partial I(z, t)}{\partial z} = Q V(z, t) + C \frac{\partial V(z, t)}{\partial t} \right]$$

These are the TL eqs. also called telegrapher's eqs

convert the TL eqs to phasor form.

$$V(z,t) = \operatorname{Re}\{V_S(z)e^{j\omega t}\}$$

$$I(z,t) = \operatorname{Re}\{I_S(z)e^{j\omega t}\}$$

$$\frac{d}{dz} \rightarrow j\omega$$

then,

$$-\frac{dV_S}{dz} \Rightarrow (R + j\omega) I_S \quad (1)$$

$$-\frac{dI_S}{dz} \Rightarrow (G + j\omega) V_S \quad (2)$$